MTH 277, Quiz #12, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the equation of the tangent plane for $z = e^x(\sin y + 1)$ at the point $\left(0, \frac{\pi}{2}, 2\right)$. Also find the equation of the normal line in parametric form at the same point.

$\nabla F = \langle e^x(\sin y + 1), e^x \cos y \rangle$ or $\langle e^x(\sin y + 1), e^x \cos y, -1 \rangle$

$\langle 2, 0, -1 \rangle$.

$2(x-0) + 0(y - \pi/2) - 1(z - 2) = 0$

$2x - z + 2 = 0$

$\boxed{z = 2x + 2}$ tangent plane

normal line

$\vec{n}(t) = (2t + 0)\hat{\imath} + (0t + \pi/2)\hat{\jmath} + (-1t + 2)\hat{k} = 2t\hat{\imath} + \pi/2\,\hat{\jmath} + (2-t)\hat{k}$

2. Find the equation of the tangent plane to the surface $\vec{r}(u, v) = 2u \cos v\, \hat{\imath} + 3u \sin v\, \hat{\jmath} + u^2 \hat{k}$ at the point (0,6,4). Also write the equation of the normal line at the same point.

$\vec{r_u} = 2\cos v\,\hat{\imath} + 3\sin v\,\hat{\jmath} + 2u\,\hat{k}$

$\vec{r_v} = -2u \sin v\,\hat{\imath} + 3u \cos v\,\hat{\jmath} + 0\hat{k}$

$\vec{r_u} \times \vec{r_v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2\cos v & 3\sin v & 2u \\ -2u\sin v & 3u\cos v & 0 \end{vmatrix} = (0 - 6u^2\cos v)\hat{\imath} - (0 + 4u^2 \sin v)\hat{\jmath} + (6u\cos^2 v + 6u\sin^2 v)\hat{k}$

$v = \pi/2$
$u = 2$

$= -6u^2 \cos v\,\hat{\imath} - 4u^2 \sin v\,\hat{\jmath} + 6u\,\hat{k}$

$\vec{r_u} \times \vec{r_v} = \langle -6(2)^2(0), -4(2)^2(1), 6(2) \rangle = \langle 0, -16, 12 \rangle$

$0(x-0) - 16(y-6) + 12(z-4) = 0$

$-16y + 96 + 12z - 48 = 0$ tangent plane

$\frac{12z}{12} = \frac{-48 + 16y}{12}$

$\boxed{z = 4/3\,y - 4}$

$\vec{n} = (0t + 0)\hat{\imath} + (-16t + 6)\hat{\jmath} + (12t + 4)\hat{k}$

$= \boxed{(6 - 16t)}\hat{\jmath} + (12t + 4)\hat{k}$

normal line