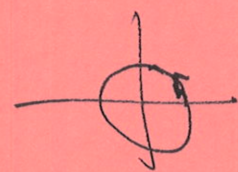
277 Homework #4 Key

1. See attached

2 a = i b = vi c = iv d = iii e = v f = ii

3a. $r(t) = 3\cos t\,\hat{\imath} + 3\sin t\,\hat{\jmath}$ $[0, 2\pi]$

$r'(t) = -3\sin t\,\hat{\imath} + 3\cos t\,\hat{\jmath}$



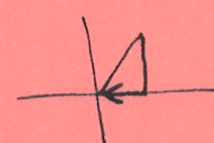
$$A = \frac{1}{2}\int_0^{2\pi} 3\cos t \cdot 3\cos t\,dt - 3\sin t(-3\sin t)\,dt =$$

$$\frac{1}{2}\int_0^{2\pi} 9(\cos^2 t + \sin^2 t)\,dt = \frac{1}{2}\int_0^{2\pi} 9\,dt = \frac{1}{2}\cdot 9t\Big|_0^{2\pi} = \frac{1}{\cancel{2}}9(\cancel{2}\pi) = 9\pi$$

b. $\vec{r}_1(t) = 5t\,\hat{\imath} + 4t\,\hat{\jmath}$ $[0,1]$ $\vec{r}_1{}'(t) = 5\hat{\imath} + 4\hat{\jmath}$

$\vec{r}_2(t) = 5\hat{\imath} + (4-4t)\hat{\jmath}$ $[0,1]$ $\vec{r}_2{}'(t) = 0\hat{\imath} - 4\hat{\jmath}$

$\vec{r}_3(t) = (5-5t)\hat{\imath} + 0\hat{\jmath}$ $[0,1]$ $\vec{r}_3{}'(t) = -5\hat{\imath} + 0\hat{\jmath}$



$$A = \frac{1}{2}\int_0^1 \underbrace{5t(4) - 4t(5)}_{=0}\,dt + \frac{1}{2}\int_0^1 5(-4)\,dt - \underbrace{(4-4t)(0)}_{0} + \frac{1}{2}\int_0^1 \underbrace{(5-5t)0 - 0(-5)}_{=0}\,dt$$

$$= \frac{1}{2}\int_0^1 -20\,dt = -10\int_0^1 dt = -10t\Big|_0^1 = -10 \rightarrow \text{Area} = 10.$$

c. $\vec{r}_1(t) = t\,\hat{\imath} + t^2\,\hat{\jmath}$ $[0,2]$ $\vec{r}_1{}'(t) = \hat{\imath} + 2t\,\hat{\jmath}$

$\vec{r}_2(t) = (2-2t)\hat{\imath} + 4\hat{\jmath}$ $[0,1]$ $\vec{r}_2{}'(t) = -2\hat{\imath} + 0\hat{\jmath}$

$\vec{r}_3(t) = 0\hat{\imath} + (4-4t)\hat{\jmath}$ $[0,1]$ $\vec{r}_3{}'(t) = 0\hat{\imath} - 4\hat{\jmath}$

$$A = \frac{1}{2}\int_0^2 t(2t) - t^2(1)\,dt + \frac{1}{2}\int_0^1 (2-2t)(0) - 4(-2)\,dt + \frac{1}{2}\int_0^1 \underbrace{0(-4) - (4-4t)0}_{=0}\,dt$$

$$= \frac{1}{2}\int_0^2 t^2\,dt + \frac{1}{2}\int_0^1 8\,dt = \frac{1}{2}\cdot\frac{1}{3}t^3\Big|_0^2 + \frac{1}{2}\cdot 8t\Big|_0^1 = \frac{1}{6}(8) + 4 = 16/3$$

4 a. $\int_C 3(x-y)\,ds$ $\vec{r}(t) = t\,\hat{\imath} + (2-t)\hat{\jmath}$ $[0,2]$

$r'(t) = \hat{\imath} - \hat{\jmath}$

$\|r'(t)\| = \sqrt{1^2 + 1^2} = \sqrt{2}$.

$$\int_0^2 3(t - 2 + t)\sqrt{2}\,dt = 3\sqrt{2}\int_0^2 2t - 2\,dt = 3\sqrt{2}\left[t^2 - 2t\right|_0^2 = 3\sqrt{2}(4-4) = 0$$

b. $\int kz\,ds$ $\vec{r}(t) = t^2\hat{\imath} + 2t\hat{\jmath} + t\hat{k}$ $[1,3]$

$\vec{r}\,'(t) = 2t\hat{\imath} + 2\hat{\jmath} + 1\hat{k}$

$\|r'(t)\| = \sqrt{4t^2 + 4 + 1} = \sqrt{4t^2 + 5}$

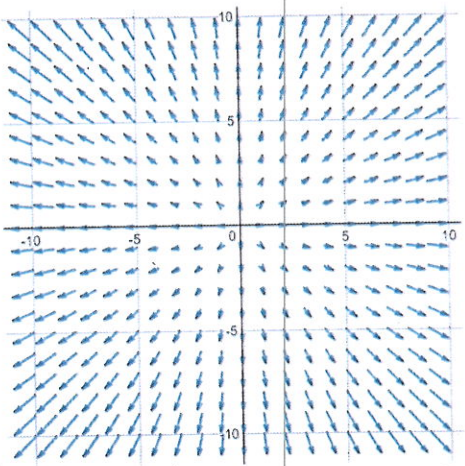
$u = 4t^2 + 5$ $t = 1 \rightarrow 9$, $t = 3 \rightarrow 41$

$du = 8t\,dt$

$\frac{1}{8}du = t\,dt$

$$k\int_1^3 t\sqrt{4t^2+5}\,dt = k\int_9^{41} \frac{1}{8}u^{1/2}\,du = \frac{k}{\cancel{8}_4}\cdot\frac{\cancel{2}}{3}u^{3/2}\Big|_9^{41} = \frac{k}{12}(41\sqrt{41} - 27)$$

1a.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-10
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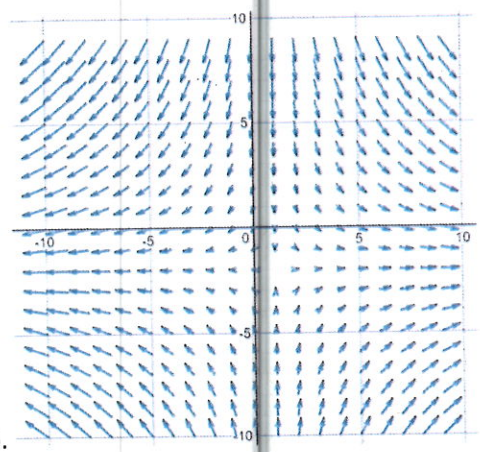
b.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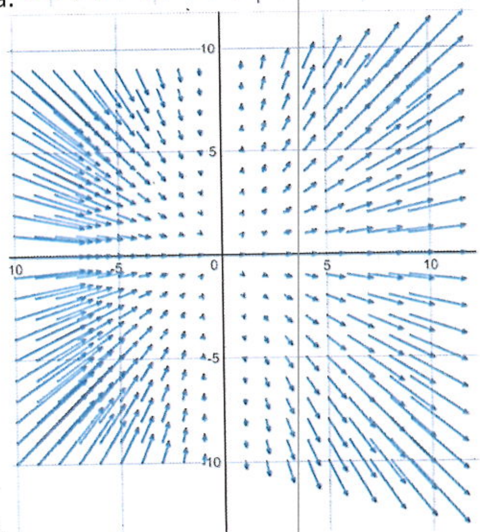
c.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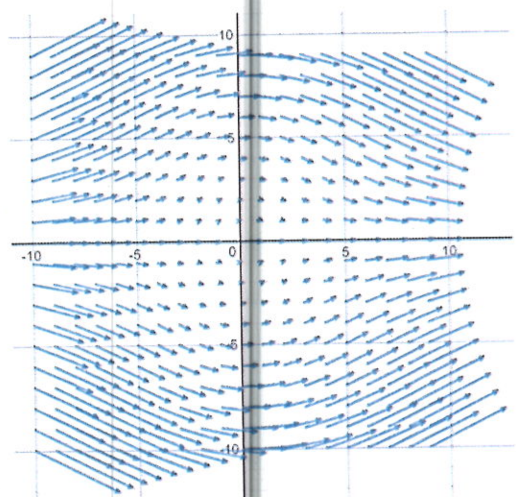
d.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-5
-10

4c. $\int_c \vec{F}\cdot d\vec{r}$ $\quad \vec{F} = x\hat{\imath} + y\hat{\jmath}$ $\quad \vec{r}(t) = t\hat{\imath} + t\hat{\jmath}$ $\quad [0,1]$

$\langle t\hat{\imath} + t\hat{\jmath}\rangle \cdot \langle r'(t) = \hat{\imath} + \hat{\jmath}\rangle$

$\int_0^1 t + t\,dt = \int_0^1 2t\,dt = t^2 \Big|_0^1 = 1$

d. $\int \vec{F}\cdot d\vec{r}$ $\quad F = yz\hat{\imath} + xz\hat{\jmath} + xy\hat{k}$ $\quad \vec{r}(t) = 5t\hat{\imath} + 3t\hat{\jmath} + 2t\hat{k}$ $\quad [0,1]$

$F = \langle 6t^2, 10t^2, 15t^2\rangle \cdot \quad r'(t) = \langle 5, 3, 2\rangle$

$\int_0^1 30t^2 + 30t^2 + 30t^2\,dt = \int_0^1 90t^2\,dt = 30t^3 \Big|_0^1 = 30$

e. $\int (x^2+y^2)\,dx + 2(xy)\,dy$ $\quad \vec{r}_1(t) = t^3\hat{\imath} + t^2\hat{\jmath}$ $\quad [0,2]$

$\vec{r}_2(t) = 2\cos t\,\hat{\imath} + 2\sin t\,\hat{\jmath}$ $\quad [0, \pi/2]$

$\vec{r}_1'(t) = 3t^2\hat{\imath} + 2t\hat{\jmath}$

$\vec{r}_2'(t) = -2\sin t\,\hat{\imath} + 2\cos t\,\hat{\jmath}$

$\int_0^2 (t^6 + t^4)3t^2 + 2t^3t^2\,2t\,dt + \int_0^{\pi/2} 4\cdot(-2\sin t) + 2\cdot 2\cos t\cdot 2\sin t\,2\cos t\,dt$

$= \int_0^2 3t^8 + 3t^6 + 4t^6\,dt + \int_0^{\pi/2} -8\sin t + 16\cos^2 t\sin t\,dt$

$\int_0^2 3t^8 + 7t^6\,dt + \int_0^{\pi/2} -8\sin t - 16\cos^2 t\,(-\sin t)\,dt$

$\frac{1}{3}t^9 + t^7 \Big|_0^2 + 8\cos t + \frac{16}{3}\cos^3 t \Big|_0^{\pi/2} = \frac{512}{3} + 128 + 8(0) + \frac{16}{3}(0) - 8 - \frac{16}{3}(1)$

$= \frac{856}{3}$

5a. $r(t) = 6t\hat{\imath} - 7t^2\hat{\jmath} + t^3\hat{k}$, $a = 1$

$r'(t) = 6\hat{\imath} - 14t\hat{\jmath} + 3t^2\hat{k}$ $\quad \int r(t)\,dt = (3t^2 + C_1)\hat{\imath} + (-\frac{7}{3}t^3 + C_2)\hat{\jmath} + (\frac{1}{4}t^4 + C_3)\hat{k}$

$r''(t) = 0\hat{\imath} - 14\hat{\jmath} + 6t\hat{k}$ $\quad \int_0^1 r(t)\,dt = 3\hat{\imath} - \frac{7}{3}\hat{\jmath} + \frac{1}{4}\hat{k}$

$\|r'(t)\| = \sqrt{36 + 196t^2 + 9t^4} = (36 + 196t^2 + 9t^4)^{1/2}$

$D_t\|r'(t)\| = \frac{1}{2}(36 + 196t^2 + 9t^4)^{-1/2}(392t + 36t^3)$

$\int_1^2 \|r'(t)\|\,dt \approx 23.0008\ldots$

b. $r(t) = (\sin t - t\cos t)\hat{\imath} + (\cos t + t\sin t)\hat{\jmath} + t^2\hat{k}$ $\quad a = \pi/2$

$r'(t) = (\cos t - \cos t + t\sin t)\hat{\imath} + (-\sin t + \sin t + t\cos t)\hat{\jmath} + 2t\hat{k}$

$= t\sin t\,\hat{\imath} + t\cos t\,\hat{\jmath} + 2t\hat{k}$

56 cont'd

$r''(t) = (\sin t + t\cos t)\hat{i} + (\cos t - t\sin t)\hat{j} + 2\hat{k}$

$\int r(t)\,dt = (-t\sin t + C_1)\hat{i} + (-t\cos t + C_2)\hat{j} + (\frac{1}{3}t^3 + C_3)\hat{k}$

$\int_0^{\pi/2} r(t)\,dt = -\pi/2\,\hat{i} + 0\hat{j} + \frac{\pi^3}{24}\hat{k}$

$\|r'(t)\| = \sqrt{t^2\sin^2 t + t^2\cos^2 t + 4t^2} = \sqrt{t^2(\sin^2 t + \cos^2 t) + 4t^2} = \sqrt{t^2+4t^2} = \sqrt{5t^2}$
$= \sqrt{5}\,t$

$D_t\|r'(t)\| = \sqrt{5}$

$\int_1^2 \sqrt{5}\,t\,dt = \frac{\sqrt{5}}{2}t^2\Big|_1^2 = \frac{\sqrt{5}}{2}(4-1) = \frac{3\sqrt{5}}{2}$

c. $r(t) = e^t\hat{i} + \sec^2 t\,\hat{j} + \frac{1}{t^2+1}\hat{k}$ $a=1$

$r'(t) = e^t\hat{i} + 2\sec^2 t\tan t\,\hat{j} + \frac{-2t}{(t^2+1)^2}\hat{k}$

$r''(t) = e^t\hat{i} + (4\sec^2 t\tan^2 t + 2\sec^3 t)\hat{j} + \left(\frac{-2(t^2+1)^2 - (-2t)(t^2+1)(2)(2t)}{(t^2+1)^4}\right)\hat{k}$

$\int r(t) = (e^t + C_1)\hat{i} + (\tan t + C_2)\hat{j} + (\arctan t + C_3)\hat{k}$

$\int_0^1 r(t)\,dt = (e-1)\hat{i} + (\tan 1)\hat{j} + (\pi/4)\hat{k}$

$\|r'(t)\| = \sqrt{e^{2t} + 4\sec^4 t\tan^2 t + \frac{4t^2}{(t^2+1)^4}} = \left(e^{2t} + 4\sec^4 t\tan^2 t + \frac{4t^2}{(t^2+1)^4}\right)^{1/2}$

$D_t\|r'(t)\| =$
$\frac{1}{2}\left(e^{2t} + 4\sec^4 t\tan^2 t + \frac{4t^2}{(t^2+1)^4}\right)^{-1/2}\left(2e^{2t} + 16\sec^4 t\tan^3 t + 8\sec^6 t\tan t + \frac{8t(t^2+1)^4 - 4t^2\cdot 4(t^2+1)^3 2t}{(t^2+1)^8}\right)$

$\int_1^2 \sqrt{e^{2t} + 4\sec^4 t\tan^2 t + \frac{4t^2}{(t^2+1)^4}}\,dt \approx 4.6847\ldots$

d. $r(t) = 4\sqrt{t}\,\hat{i} + t^2\sqrt{t}\,\hat{j} + \ln t^2\,\hat{k}$ $a=4$

$4t^{1/2}\hat{i} + t^{5/2}\hat{j} + 2\ln t\,\hat{k}$

$r'(t) = \frac{2}{\sqrt{t}}\hat{i} + \frac{5}{2}t^{3/2}\hat{j} + \frac{2}{t}\hat{k}$

$r''(t) = t^{-3/2}\hat{i} + \frac{15}{4}t^{1/2}\hat{j} - \frac{2}{t^2}\hat{k}$

$\int r(t)\,dt = (\frac{8}{3}t^{3/2} + C_1)\hat{i} + (\frac{2}{7}t^{7/2} + C_2)\hat{j}\ (2t\ln t - 2t + C_3)\hat{k}$

$\int_0^4 r(t)\,dt = \frac{64}{3}\hat{i} + \frac{256}{7}\hat{j}$ + undefined $\hat{k}$ term at 0
integral does not converge

$\|r'(t)\| = \sqrt{\frac{4}{t} + \frac{25}{4}t^3 + \frac{4}{t^2}}$ $D_t\|r'(t)\| = \frac{1}{2}\left(\frac{4}{t} + \frac{25}{4}t^3 + \frac{4}{t^2}\right)^{-1/2}\cdot\left(\frac{-4}{t^2} + \frac{75}{4}t^2 - \frac{8}{t^3}\right)$

5d cont'd

$$\int_1^2 \|r'(t)\|\,dt = \int_1^2 \sqrt{\frac{4}{t} + \frac{25}{4}t^3 + \frac{4}{t^2}}\,dt \approx 5.206$$

e. $r(t) = \frac{1}{2}\hat{\imath} + \hat{\jmath} - \hat{k}$

$r'(t) = \vec{0}$ $\quad \|r'(t)\| = 0 \quad D_t\|r'(t)\| = 0$

$r''(t) = \vec{0}$ $\quad \int_1^2 \|r'(t)\|\,dt = 0$

$\int r(t)\,dt = (\frac{1}{2}t + C_1)\hat{\imath} + (t + C_2)\hat{\jmath} + (-t + C_3)\hat{k}$

$\int_0^a r(t)\,dt = \frac{1}{2}a\hat{\imath} + a\hat{\jmath} - a\hat{k}$

6a. $f(x,y) = x^2 - 3y^2 + 7$

$f_x = 2x \rightarrow -2$

$f_y = -6y \rightarrow -6$

$(0,0)$

7a. $f_{xx} = 2$

$f_{yy} = -6$

$f_{xy} = f_{yx} = 0$

6b. $f(x,y) = \dfrac{xy}{x^2+y^2}$

$(0,0)$
$(-1,1)$
$(1,-1)$

$f_x = \dfrac{y(x^2+y^2) - xy(2x)}{(x^2+y^2)^2} \rightarrow \dfrac{0}{4}$ $\quad f_y = \dfrac{x(x^2+y^2) - xy(2y)}{x^2+y^2} \rightarrow 0$

c. $z = \sinh(2x+3y)$

$f_x = \cosh(2x+3y)\cdot 2 \rightarrow \approx 3.086$

$f_y = \cosh(2x+3y)\cdot 3 \approx 4.629$

never 0

7c. $f_{xx} = \sinh(2x+3y)\cdot 4$

$f_{yy} = \sinh(2x+3y)\cdot 9$

$f_{xy} = \sinh(2x+3y)\cdot 6 = f_{yx}$

d. $f(x,y) = \arctan\left(\frac{y}{x}\right)$

$f_x = \dfrac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\dfrac{-y}{x^2} \rightarrow -\frac{1}{2}$ $\quad f_y = \dfrac{1}{1+(y/x)^2}\cdot\dfrac{1}{x} \rightarrow -\frac{1}{2}$

$(0,0)$

e. $f(x,y,z) = \dfrac{3xz}{x+y}$

$f_x = \dfrac{3z(x+y) - 1(3xz)}{(x+y)^2} \rightarrow \dfrac{-6}{1}$ $\quad f_y = -3zx(x+y)^{-2} \rightarrow 0$ $\quad f_z = \dfrac{3x}{x+y} \rightarrow 0$

f. $f(x,y,z) = \dfrac{1}{\sqrt{1-x^2-y^2-z^2}} = (1-x^2-y^2-z^2)^{-1/2}$

$f_x = -\frac{1}{2}(1-x^2-y^2-z^2)^{-3/2}(-2x)$ $\quad f_y = -\frac{1}{2}(1-x^2-y^2-z^2)^{-3/2}(-2y)$

not defined at $(0,1,-2)$ $\quad f_z = -\frac{1}{2}(1-x^2-y^2-z^2)^{-3/2}(-2z)$

6g. $z = xe^{x/y}$

$z_x = e^{x/y} + xe^{x/y}\cdot\frac{1}{y} \to 0$ $\quad f_y = xe^{x/y}\left(-\frac{x}{y^2}\right) \to e^{-1}$

never both zero

h. $z = e^y \sin xy$

$z_x = e^y \cos(xy)\cdot y \to e\cos(-1)$ $\quad z_y = e^y \sin(xy) + e^y\cos(xy)\cdot x \to e\sin(-1) - e\cos(-1)$

$(0,0)$

i. $f(x,y) = \tanh(xy^2)$

$y=0$

$f_x = -\text{sech}^2(xy^2)\cdot y^2 \to -.64805$ $\quad f_y = -\text{sech}^2(xy^2)\cdot 2xy \to \approx 1.2961$

j. $f(x,y) = x^2 + 4xy + y^2 - 4x + 16y + 3$

$f_x = 2x + 4y - 4 \to -2$

$f_y = 4x + 2y + 16 \to 14$ $\quad (-6,4)$

7j. $f_{xx} = 2$

$f_{yy} = 2$

$f_{xy} = 4 = f_{yx}$

k. $w = 3x^2y - 5xyz + 10yz^2$

$w_x = 6xy - 5yz \to 10$ $\quad w_y = 3x^2 - 5xz + 10z^2 \to 40$ $\quad w_z = -5xy + 20yz \to -40$

7k. $w_{xx} = 6y$ $\quad w_{yy} = 0$ $\quad w_{zz} = 20y$

$w_{xy} = 6x - 5z = w_{yx}$ $\quad w_{xz} = -5y = w_{zx}$ $\quad w_{yz} = -5x + 20z = w_{zy}$

l. $z = \dfrac{e^x}{x+y^2}$ $\quad z_x = \dfrac{e^x(x+y^2) - e^x(1)}{(x+y^2)^2}$ $\quad z_y = \dfrac{e^x}{(x+y^2)^2}(-2y)$

8. $f(x,y) = x^2 + 4xy + y^2 - 4x + 16y + 3$

$$\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2 + 4(x+\Delta x)y + y^2 - 4(x+\Delta x) + 16y + 3 - (x^2+4xy+y^2-4x+16y+3)}{\Delta x}$$

$$\lim_{\Delta x\to 0} \frac{\cancel{x^2} + 2\Delta x\cdot x + \Delta x^2 + \cancel{4xy} + 4\Delta x y + \cancel{y^2} - \cancel{4x} - 4\Delta x + \cancel{16y} + \cancel{3} - \cancel{y^2} - \cancel{4xy} - \cancel{x^2} + \cancel{4x} - \cancel{16y} - \cancel{3}}{\Delta x}$$

$$\lim_{\Delta x\to 0} \frac{2\Delta x\cdot x + \Delta x^2 + 4\Delta x\cdot y - 4\Delta x}{\Delta x} = \lim_{\Delta x\to 0} \frac{\cancel{\Delta x}(2x + \Delta x + 4y - 4)}{\cancel{\Delta x}} = 2x + 4y - 4$$

$$\lim_{\Delta y\to 0} \frac{x^2 + 4x(y+\Delta y) + (y+\Delta y)^2 - 4x + 16(y+\Delta y) + 3 - (x^2+4xy+y^2-4x+16y+3)}{\Delta y}$$

$$\lim_{\Delta y\to 0} \frac{\cancel{x^2} - \cancel{4xy} + 4x\Delta y + \cancel{y^2} + 2\Delta y\cdot y + \Delta y^2 - \cancel{4x} + \cancel{16y} + 16\Delta y + \cancel{3} - \cancel{x^2} - \cancel{4xy} - \cancel{y^2} + \cancel{4x} - \cancel{16y} - \cancel{3}}{\Delta y}$$

$$\lim_{\Delta y\to 0} \frac{4x\Delta y + 2\Delta y\, y + \Delta y^2 + 16\Delta y}{\Delta y} = \lim_{\Delta y\to 0} \frac{\cancel{\Delta y}(4x + 2y + \Delta y + 16)}{\cancel{\Delta y}} = 4x + 2y + 16$$

9. a. $z = x^2 - y^2$

$z_x = 2x$ $\quad z_y = -2y$ $\quad z_{xx} + z_{yy} = 2 - 2 = 0$ ✓

$z_{xx} = 2$ $\quad z_{yy} = -2$

b. $z = e^y \sin x$ $\quad z_y = e^y \sin x$ $\quad z_{xx} + z_{yy} = -e^y \sin x + e^y \sin x = 0$ ✓

$z_x = e^y \cos x$ $\quad z_{yy} = e^y \sin x$

$z_{xx} = -e^y \sin x$

c. $z = \dfrac{y}{x^2+y^2}$

$z_x = -y(x^2+y^2)^{-2}(2x)$

$z_{xx} = +2y(x^2+y^2)^{-3}(2x)^2 - y(x^2+y^2)^{-2}(2)$

$z_y = (x^2+y^2)^{-1} + (-1)y(x^2+y^2)^{-2}(2y)$

$z_{yy} = -1(x^2+y^2)^{-2}(2y) + 2y(x^2+y^2)^{-3}(2y)(2y) - \cancel{4y}\,(x^2+y^2)^{-2}$

$z_{xx} + z_{yy} = 0$

10 a. $\vec{r}(u,v) = u\cos v\,\hat{i} + u\sin v\,\hat{j} + v\hat{k}$

$\vec{r}_u = \cos v\,\hat{i} + \sin v\,\hat{j} + 0\hat{k}$

$\vec{r}_v = -u\sin v\,\hat{i} + u\cos v\,\hat{j} + \hat{k}$

b. $\vec{r}(u,v) = \sin v\,\hat{i} + \cos u \sin 2v\,\hat{j} + \sin u \sin 2v\,\hat{k}$

$\vec{r}_u = 0\hat{i} - \sin u \sin 2v\,\hat{j} + \cos u \sin 2v\,\hat{k}$

$\vec{r}_v = \cos v\,\hat{i} + 2\cos u \cos 2v\,\hat{j} + 2\sin u \cos 2v\,\hat{k}$

c. $\vec{r}(u,v) = (1-u)(3+\cos v)\cos(4\pi u)\hat{i} + (1-u)(3+\cos v)(\sin 4\pi u)\hat{j} +$
$(3u + (1-u)\sin v)\hat{k}$

$\vec{r}_u = \left[(-1)(3+\cos v)\cos 4\pi u + (1-u)(3+\cos v)(-4\pi \sin 4\pi u)\right]\hat{i} +$

$\left[-(1)(3+\cos v)(\sin 4\pi u) + (1-u)(3+\cos v)4\pi\cos 4\pi u\right]\hat{j} +$

$(3 - \sin v)\hat{k}$

$\vec{r}_v = \left[(1-u)\cos 4\pi u(-\sin v)\right]\hat{i} + \left[(1-u)\sin 4\pi u(-\sin v)\right]\hat{j} + (1-u)\cos v\,\hat{k}$

10d. $\vec{r}(u,v) = (u^2+1)\hat{i} + (v^3+1)\hat{j} + (u+v)\hat{k}$

$\vec{r}_u = 2u\hat{i} + 0\hat{j} + \hat{k}$

$\vec{r}_v = 0\hat{i} + 3v^2\hat{j} + \hat{k}$

11 a. $f(x,y) = x^2y^{-1} = \frac{x^2}{y}$ $\Delta x = .05$

$\Delta y = -0.1$

$f_x = 2xy^{-1} = \frac{2x}{y}$ $f_y = \frac{-x^2}{y^2}$

$f_x(1,2) = \frac{2(1)}{2} = 1$ $f_y(1,2) = \frac{-(1)^2}{4} = -\frac{1}{4}$

$f(1,2) = \frac{1}{2}$

$f(1.05, 1.9) \approx \frac{1}{2} + (1)(.05) + (-\frac{1}{4})(-.1) = \frac{23}{40} = .575$

b. $g(x,y,z) = x^2yz^2 + \sin yz$ $g(1.05, 2.1, -.01)$ $\Delta x = .05$

$\Delta y = .1$

$\Delta z = -.01$

$g_x = 2xyz^2 \to 0$ $g(1,2,0) = 0+0$

$g_y = x^2z^2 + z\cos yz \to 0$

$g_z = x^2y2z + y\cos yz \to 2$

$g(1.05, 2.1, -.01) \approx 0 + 0(.05) + (0)(.1) + 2(-.01) = -.02$

c. $f(x,y) = xe^y$ $f(1,2) = e^2$ $\Delta x = .05$

$\Delta y = -.1$

$f_x = e^y \to e^2$

$f_y = xe^y \to e^2$

$f(1.05, 1.9) \approx e^2 + e^2(.05) + e^2(-.1) \approx 7.0196$

$.95e^2$

d. $g(x,y,z) = \frac{x+y}{z-2y}$ $g(1,2,0) = \frac{1+2}{0-2(2)} = \frac{3}{-4}$ $\Delta x = .05$

$\Delta y = .1$

$\Delta z = -.01$

$g_x = \frac{1}{z-2y} \to \frac{1}{-4}$

$g_y = \frac{1(z-2y) - (-2)(x+y)}{(z-2y)^2} \to \frac{1(0-4) - (-2)(3)}{(-4)^2} = \frac{2}{16} = \frac{1}{8}$

$g_z = (x+y)(z-2y)^{-2} \to 3(-4)^{-2} = \frac{3}{16}$

$g(1.05, 2.1, -.01) \approx -\frac{3}{4} + (-1/4)(.05) + (\frac{1}{8})(.1) + (\frac{3}{16})(-.01) = -.751875$