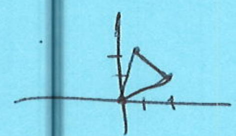
277 Homework #11 Key

1. $J = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 4 - 1 = 3$

$x - 3y = (2u+v) - 3(u+2v) = 2u + v - 3u - 6v = -u - 5v$

$y = \frac{1}{2}x \qquad y = 2x$



$m = \frac{2-1}{1-2} = \frac{1}{-1} = -1$

$y - 1 = -1(x-2) \rightarrow \underset{+1}{y-1} = \underset{+1}{-x+2} \rightarrow y = -x + 3$

$u + 2v = -(2u+v) + 3$

$u + 2v = -2u - v + 3$

$\frac{3u + 3v = 3}{3} \rightarrow u + v = 1 \qquad v = 1 - v$

$\int_0^1 \int_0^{1-v} -u - 5v \, du\, dv =$

$\int_0^1 -\frac{u^2}{2} - 5uv \Big|_0^{1-u} dv = \int_0^1 -\frac{\overset{1-2v+v^2}{(1-v)^2}}{2} - 5(1-v)v \, dv =$

$\int_0^1 -\frac{1}{2} + v - \frac{1}{2}v^2 - 5v + 5v^2 \, dv = \int_0^1 -\frac{1}{2} - 4v + \frac{9}{2}v^2 \, dv = -\frac{1}{2}v - 2v^2 + \frac{3}{2}v^3 \Big|_0^1$

$= -\frac{1}{2} - 2 + \frac{3}{2} = \boxed{-1}$

2. a. $J = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = |ad - bc|$

b. $J = \begin{vmatrix} e^u \sin v & e^u \cos v \\ e^u \cos v & -e^u \sin v \end{vmatrix} = -e^{2u}\sin^2 v - e^{2u}\cos^2 v = -e^{2u}$

$|J| = e^{2u}$

c. $J = \begin{vmatrix} 2x & -1 \\ -1 & 1 \end{vmatrix} = 2x - 1$

d. $J = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta = 1$

e. $J = \begin{vmatrix} \frac{1}{v} & -\frac{u}{v^2} \\ 1 & 1 \end{vmatrix} = \frac{1}{v} + \frac{u}{v^2}$

3a. $J = \begin{vmatrix} 1-v & -u & 0 \\ v(1-w) & u(1-w) & -uv \\ vw & uw & uv \end{vmatrix} = (1-v)\left(u(1-w)uv + u^2vw\right) - (-u)\left(v(1-w)uv + uv^2w\right)$

$+ 0\left(v(1-w)uw - uvw(1-w)\right)$

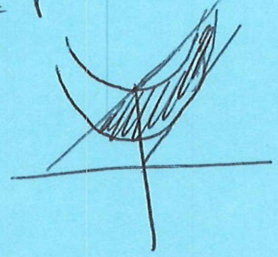
$= (1-v)\left(u^2v - u^2\cancel{v}w + u^2\cancel{v}w\right) + u\left(uv^2 - uv^2\cancel{w} + u\cancel{v^2}w\right) + 0$

$= u^2v - u^2\cancel{v^2} + u^2\cancel{v^2} = u^2v$

b. $J = \begin{vmatrix} 1 & -1 & 1 \\ 2v & 2u & 0 \\ 1 & 1 & 1 \end{vmatrix} = 1(2u-0) + 1(2v-1) + 1(2v-2u) =$

$2\cancel{u} + 2v + 2v - \cancel{2u} = 4v$

c. $J = \begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r$



4. a. $y - x^2 = 1$ $y - x = 0$ $y = v + x$

$y - x^2 = 4$ $y - x = 4$ $u = v + x - x^2$

$u = y - x^2$ $v = y - x$

$u - v = x - x^2$

$v - u = x^2 - x + \frac{1}{4}$

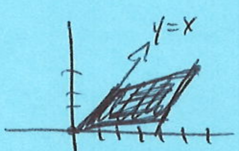
$+\frac{1}{4}$

$v - u + \frac{1}{4} = (x - \frac{1}{2})^2$

$\pm\sqrt{v - u + \frac{1}{4}} + \frac{1}{2} = x$

$y = \pm\sqrt{v - u + \frac{1}{4}} + \frac{1}{2} + v$

b.



$[0, 3]$

$u = x - y$

$v = 4y - x$ $[0, 6]$

$(0,0) \to (2,2)$

$y = x$

$\frac{3-2}{6-2} = \frac{1}{4}$

$y - 2 = \frac{1}{4}(x - 2)$

$y - 2 = \frac{1}{4}x - \frac{1}{2}$

$+2 \qquad +2$

$y = \frac{1}{4}x + \frac{3}{2}$

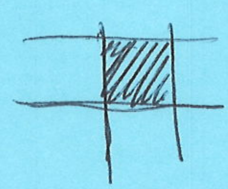
$4y - x = 6$

$\frac{1-3}{4-6} = \frac{-2}{-2} = 1$

$y - 1 = x - 4$

$y = x - 3$

$y = \frac{1}{4}x$

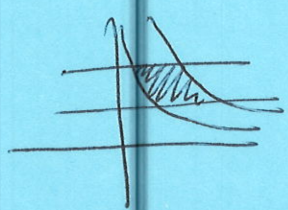


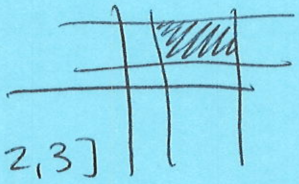
4c. $xy=1,\ xy=4,\ y=1,\ y=4$

$xy=u \quad [1,4]$

$y=v \quad [1,4]$

$x=\frac{u}{v}$





d. $y=2x \rightarrow \frac{y}{x}=2 \quad \frac{y}{x}=3 \qquad u=\frac{y}{x} \quad [2,3]$

$y-x^2=0 \qquad y-x^2=1 \qquad v=y-x^2 \quad [0,1]$

$ux=y$

$v=ux-x^2 \rightarrow$

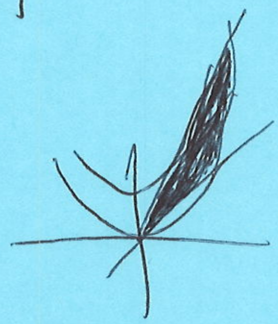
$x^2-ux=-v$

$x^2-ux+\frac{u^2}{4}=\frac{u^2}{4}-v$

$(x-\frac{u}{2})^2=\frac{u^2}{4}-v$

$x=\pm\sqrt{\frac{u^2}{4}-v}+\frac{u}{2}$

$y=\pm u\sqrt{\frac{u^2}{4}-v}+\frac{u^2}{2}$



5a. $y=0 \quad y=x$

$y=3 \quad y=x+3$

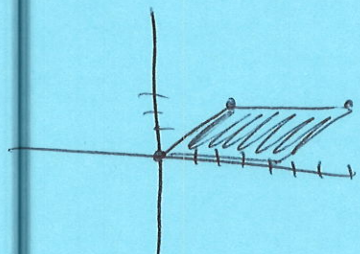
$u=y \quad [0,3]$

$J=\begin{vmatrix}1 & 1\\ 0 & 1\end{vmatrix}=1$

$x-y=0$

$x-y=-3 \quad [-3,0]$

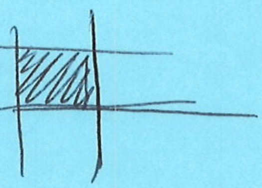
$v=x-y \qquad x=u+v$



$\int_0^3\int_{-3}^0 u\,v\,dv\,du=\int_0^3 \frac{1}{2}uv^2\Big|_{-3}^0=\int_0^3 -\frac{9}{2}u\,du$

$=-\frac{9}{4}u^2\Big|_0^3=$

$=-\frac{81}{4}$



b. $y=2x \quad y=\frac{1}{4}x$

$\frac{y}{x}=2,\ \frac{y}{x}=\frac{1}{4} \qquad [\frac{1}{4},2] \quad u=\frac{y}{x}$

$xy=1 \quad xy=4$

$v=xy \qquad [1,4]$

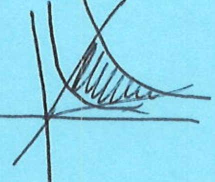
$ux=y$

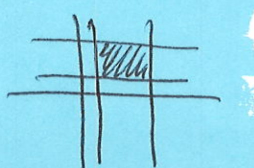
$v=xux$

$\frac{v}{u}=x^2 \qquad x=\sqrt{\frac{v}{u}}=v^{1/2}u^{-1/2}$

$y=\sqrt{uv}=u^{1/2}v^{1/2}$

$J=\begin{vmatrix}-\frac{1}{2}u^{-3/2}v^{1/2} & \frac{1}{2}v^{-1/2}u^{-1/2}\\ \frac{1}{2}u^{-1/2}v^{1/2} & \frac{1}{2}u^{1/2}v^{-1/2}\end{vmatrix}=$



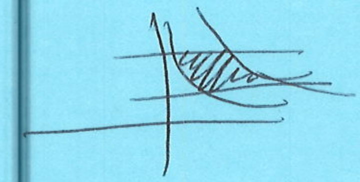


$-\frac{1}{4}u^{-1}-\frac{1}{4}u^{-1}=\frac{-1}{4u} \qquad |J|=\frac{1}{4u} \qquad \frac{1}{4}\int_{1/4}^2\int_1^4 e^{-v/2}\frac{1}{u}\,dv\,du=\frac{-1}{2}\int_{1/4}^2(e^{-1/2}-e^{-2})\,du=\frac{1}{2}(e^{-1/2}-e^{-2})\cdot(\ln 2-\ln(\frac{1}{4}))$

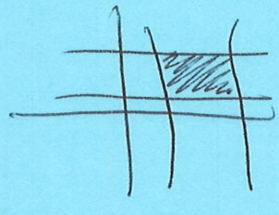
5c. $xy=1 \quad xy=4$ $\qquad y=1,\ y=4$

$u=xy \quad [1,4]$ $\qquad v=y \quad [1,4]$

$x=\frac{u}{v}$



$J=\begin{vmatrix} \frac{1}{v} & \frac{-u}{v^2} \\ 0 & 1 \end{vmatrix} = \frac{1}{v}$



$\int_1^4\int_1^4 \cancel{y} \sin u \cdot \frac{1}{\cancel{v}}\, du\, dv = \int_1^4 -\cos u \Big|_1^4 dv =$

$3(\cos 1-\cos 4)$

6.a. $r'(t)=2t\hat{i}+3t^2\hat{j}$ $\qquad 2\hat{i}+3\hat{j}$

$\|r'(t)\|=\sqrt{4t^2+9t^4}$ $\qquad \sqrt{13}$

$r''(t)=2\hat{i}+6t\hat{j}$ $\qquad 2\hat{i}+6\hat{j}$

$r'''(t)=0\hat{i}+6\hat{j}$ $\qquad 6\hat{j}$

b. $r'(t)=3\hat{i}+1\hat{j}+\frac{1}{2}t\hat{k}$

$\|r'(t)\|=\sqrt{10+\frac{t^2}{4}}$

$r''(t)=0\hat{i}+0\hat{j}+\frac{1}{2}\hat{k}$

$r'''(t)=\vec{0}$

c. $r'(t)=(1-\cos t)\hat{i}+(1+\sin t)\hat{j}$ $\qquad t=\pi$

$\qquad 2\hat{i}+1\hat{j}$

$\|r'(t)\|=\sqrt{(1-\cos t)^2+(1+\sin t)^2}$ $\qquad \sqrt{5}$

$r''(t)=\sin t\,\hat{i}+\cos t\,\hat{j}$ $\qquad 0\hat{i}-\hat{j}$

$r''(t)=\cos t\,\hat{i}-\sin t\,\hat{j}$ $\qquad (-1)\hat{i}+0\hat{j}$

d. $r'(t)=(e^t\cos t-e^t\sin t)\hat{i}+(e^t\sin t+e^t\cos t)\hat{j}+e^t\hat{k}$

$\|r'(t)\|=\sqrt{(\cos t-\sin t)^2+(\sin t+\cos t)^2+1}\ e^t$

$\underbrace{\cos^2 t-\cancel{2\sin t\cos t}+\sin^2 t}_{=1}+\underbrace{\sin^2 t+\cancel{2\sin t\cos t}+\cos^2 t}_{=1}+1=\sqrt{3}e^t$

$r''(t)=-2e^t\sin t\,\hat{i}+2e^t\cos t\,\hat{j}+e^t\hat{k}$

$r'''(t)=(-2e^t\cos t-2e^t\sin t)\hat{i}+(2e^t\cos t-2e^t\sin t)\hat{j}+e^t\hat{k}$

7a. $v(t)=\int -\cos t\,\hat{i}-\sin t\,\hat{j}\,dt=(\underbrace{-\sin t}_{=0}+C_1)\hat{i}+(\underbrace{\cos t}_{=1}+C_2)\hat{j}+C_3\hat{k}$

$C_1=0 \qquad C_2=0 \qquad C_3=1$

$r(t)=\int -\sin t\,\hat{i}+\cos t\,\hat{j}+\hat{k}\,dt=(\cos t+C_1)\hat{i}+(\sin t+C_2)\hat{j}+(t+C_3)\hat{k}$

$C_1=0 \qquad C_2=0 \qquad C_3=0$

$r(t)=\cos t\,\hat{i}+\sin t\,\hat{j}+t\hat{k}$

7b. $v(t) = \int -32\hat{k}\,dt = C_1\hat{i} + C_2\hat{j} + (-32t + C_3)\hat{k}$

$C_1 = 3 \quad C_2 = -2 \quad C_3 = 1$

$r(t) = \int 3\hat{i} - 2\hat{j} + (-32t + 1)\hat{k}\,dt = (3t + C_1)\hat{i} + (-2t + C_2)\hat{j} + (-16t^2 + t + C_3)\hat{k}$

$C_1 = 0 \quad C_2 = 5 \quad C_3 = 2$

$r(t) = (3t)\hat{i} + (-2t + 5)\hat{j} + (-16t^2 + t + 2)\hat{k}$

c. $v(t) = \int e^t\hat{i} - 8k\,dt = (e^t + C_1)\hat{i} + C_2\hat{j} + (-8t + C_3)\hat{k}$

$C_1 = 1 \quad C_2 = 3 \quad C_3 = 1$

$r(t) = \int (e^t + 1)\hat{i} + 3\hat{j} + (-8t + 1)\hat{k}\,dt = (e^t + t + C_1)\hat{i} + (3t + C_2)\hat{j} + (-4t^2 + t + C_3)\hat{k}$

$C_1 = -1 \quad C_2 = 0 \quad C_3 = 0$

$r(t) = (e^t + t - 1)\hat{i} + (3t)\hat{j} + (-4t^2 + t)\hat{k}$

8a. $\int_0^{4\pi} 4|\cos t \sin t|\sqrt{5}\,dt = 32\sqrt{5}\int_0^{\pi/2} \cos t \sin t\,dt = 16\sqrt{5}\sin^2 t\Big|_0^{\pi/2} = 16\sqrt{5}$

$r'(t) = -2\sin t\,\hat{i} + 2\cos t\,\hat{j} + 1\hat{k} \qquad \|r'(t)\| = \sqrt{4\sin^2 t + 4\cos^2 t + 1} = \sqrt{5}$

b. $r'(t) = -3\sin t\,\hat{i} + 2\hat{j} + \cos t\,\hat{k} \qquad = \sqrt{9\sin^2 t + 4 + \cos^2 t} = \sqrt{8\sin^2 t + 5}$

$1 - \cos^2 t$

$k\int_0^{\pi} \sin t\sqrt{13 - 8\cos^2 t}\,dt \qquad u = \cos t$

$du = -\sin t$

$k\left(\frac{13\sin^{-1}\left(\frac{2\sqrt{26}}{13}\right)\cdot\sqrt{2}}{4} + \sqrt{5}\right)$

c. $r'(t) = (2 - 4\cos t)\hat{i} + (4\sin t)\hat{j} \qquad \sqrt{(2 - 4\cos t)^2 + 16\sin^2 t} =$

$4 - 16\cos t + \underbrace{16\sin^2 t + 16\cos^2 t}_{=1}$

$\int_0^{\pi} \sin t\sqrt{20 - 16\cos t}\,dt = 26/3 \qquad 20 - 16\cos t$

d. $r'(t) = 2t\hat{i} + 4\sin t\cos t\,\hat{j} + (-2\csc^2 t)\hat{k}$

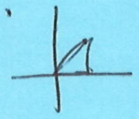
$k\int_{\pi/4}^{\pi/2} 2\sin^2 t \cdot 2\cot t\sqrt{4t^2 + 16\sin^2 t\cos^2 t + 4\csc^4 t}\,dt \approx 2.87923$

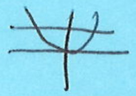
9a. $\int_{-1}^{1}\int_{-1}^{1} \sin x\cos y \cdot \cos x \cdot \sin y\,dy\,dx = 0$ orthogonal

b. $\int_{-1}^{1}\int_{-1}^{1} e^y\sin 2x \cdot e^y\sin x\,dy\,dx = \frac{1}{3}(-\sin 3 - 3\sin 1)\sinh 2$

not orthogonal

c. $\int_{-1}^{1}\int_{-1}^{1} \sinh x\cosh y\,dy\,dx = 0$ orthogonal

10a.  $A = \frac{1}{2}(1)(1)$ $\quad \frac{1}{(1/2)}\int_0^1\int_0^x \sin(x+y)\,dy\,dx = -\sin 2 + 2\sin 1$

b.  $\int_{-2}^{2}\int_{x^2}^{4} dy\,dx = A = \frac{32}{3}$ $\quad \frac{3}{32}\int_{-2}^{2}\int_{x^2}^{4} \sin^2 x\,dy\,dy =$

$\frac{3}{32}\left(\frac{1}{6}\right)\left(12\cos^2 2 - 3\sin(2)\cos 2 + 26\right)$

c.  $\int_0^{2\pi}\int_0^{2+\cos\theta} r\,dr\,d\theta = A = \frac{9\pi}{2}$

$\frac{2}{9\pi}\int_0^{2\pi}\int_0^{2+\cos\theta} \cosh(r^2)\,r\,dr\,d\theta = \frac{2}{9\pi}(2180.65) \approx 154.25$

11a. $M = \int_0^{\pi/2}\int_0^a kr^2 \cdot r\,dr\,d\theta = \frac{k\pi a^4}{8}$

$M_x = \int_0^{\pi/2}\int_0^a kr^2 \cdot r\sin\theta\, r\,dr\,d\theta = \frac{ka^5}{5}$

$M_y = \int_0^{\pi/2}\int_0^a kr^2 \cdot r\cos\theta \cdot r\,dr\,d\theta = \frac{ka^5}{5}$

$\bar{x} = \frac{M_y}{M} = \cancel{\frac{k\pi a^4}{8}} \cdot \frac{5}{\cancel{ka^5}} = \frac{5\pi}{8a}$ $\qquad \left(\frac{5\pi}{8a}, \frac{5\pi}{8a}\right)$

$\bar{y} = \frac{M_x}{M} = \frac{5\pi}{8a}$

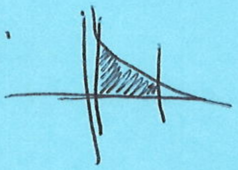
b. $M = \int_0^{L/2}\int_0^{\cos(\pi x/L)} k\,dy\,dx = \frac{kL}{\pi}$

$M_x = \int_0^{L/2}\int_0^{\cos\left(\frac{\pi x}{L}\right)} ky\,dy\,dx = \frac{kL}{8}$

$M_y = \int_0^{L/2}\int_0^{\cos\left(\frac{\pi x}{L}\right)} kx\,dy\,dx = kL^2\left(\frac{1}{2\pi} - \frac{1}{\pi^2}\right)$

$\bar{x} = \frac{M_y}{M} = \frac{kL^2\left(\frac{1}{2\pi} - \frac{1}{\pi^2}\right)}{\cancel{kL}} \cdot \pi = L\left(\frac{1}{2} - \frac{1}{\pi}\right)$

$\bar{y} = \frac{M_x}{M} = \frac{\cancel{kL}}{8} \cdot \frac{\pi}{\cancel{kL}} = \frac{\pi}{8}$ $\qquad \left(L\left(\frac{1}{2} - \frac{1}{\pi}\right), \frac{\pi}{8}\right)$

11c. 

$$M=\int_1^4\int_0^{4/x} kx^2\,dy\,dx = 30k$$

$$M_x=\int_1^4\int_0^{4/x} kx^2 y\,dy\,dx = 24k$$

$$M_y=\int_1^4\int_0^{4/x} kx^3\,dy\,dx = 84k$$

$$\bar{x}=\frac{M_y}{M}=\frac{84\cancel{k}}{30\cancel{k}}=\frac{14}{5} \qquad \left(\frac{14}{5},\frac{4}{5}\right)$$

$$\bar{y}=\frac{M_x}{M}=\frac{24k}{30\cancel{k}}=\frac{4}{5}$$

d. $$\int_0^{2\pi}\int_0^{1+\cos\theta} kr\,dr\,dr = \frac{3k\pi}{2} = M$$

$$M_y=\int_0^{2\pi}\int_0^{1+\cos\theta} kr\cos\theta\, r\,dr\,d\theta = \frac{5k\pi}{4}$$

$$M_x=\int_0^{2\pi}\int_0^{1+\cos\theta} kr\sin\theta\, r\,dr\,d\theta = 0$$

$$\bar{x}=\frac{M_y}{M}=\frac{5\cancel{k\pi}}{\cancel{4}_2}\cdot\frac{\cancel{2}}{3\cancel{k\pi}}=\frac{5}{6} \qquad \left(\frac{5}{6},0\right)$$

$$y=\frac{M_x}{M}=0$$

12a. $$M=\int_0^4\int_0^4\int_0^{4-x} ky\,dz\,dy\,dx = 64k$$

$$M_{xy}=\int_0^4\int_0^4\int_0^{4-x} kyz\,dz\,dy\,dx=\frac{256}{3}k$$

$$M_{xz}=\int_0^4\int_0^4\int_0^{4-x} ky^2\,dz\,dy\,dx=\frac{512}{3}k$$

$$M_{yz}=\int_0^4\int_0^4\int_0^{4-x} kxy\,dz\,dy\,dx=\frac{256}{3}k$$

$$\bar{x}=\frac{M_{yz}}{M}=\frac{\overset{4}{\cancel{256k}}}{3}\cdot\frac{1}{\cancel{64k}}=\frac{4}{3} \qquad \left(\frac{4}{3},\frac{8}{3},\frac{4}{3}\right)$$

$$\bar{y}=\frac{M_{xz}}{M}=\frac{\overset{8}{\cancel{512k}}}{3}\cdot\frac{1}{\cancel{64k}}=\frac{8}{3}$$

$$\bar{z}=\frac{M_{xy}}{M}=\frac{\overset{4}{\cancel{256k}}}{3}\cdot\frac{1}{\cancel{64k}}=\frac{4}{3}$$

12b. $M = \int_{-2}^{2}\int_0^1\int_0^{\frac{1}{y^2+1}} kz\,dz\,dy\,dx = \frac{\pi+2}{4}k$

$M_{xy} = \int_{-2}^{2}\int_0^1\int_0^{\frac{1}{y^2+1}} kz^2\,dz\,dy\,dx = \frac{3\pi+8}{24}k$

$M_{yz} = \int_{-2}^{2}\int_0^1\int_0^{\frac{1}{y^2+1}} kxz\,dz\,dy\,dx = 0$

$M_{xz} = \int_{-2}^{2}\int_0^1\int_0^{\frac{1}{y^2+1}} kyz\,dz\,dy\,dx = \frac{1}{2}k$

$\bar{x} = \frac{M_{yz}}{M} = 0$

$\bar{y} = \frac{M_{xz}}{M} = \frac{\cancel{k}}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{(\pi+2)\cancel{k}} = \frac{2}{\pi+2}$

$\bar{z} = \frac{M_{xy}}{M} = \frac{(3\pi+8)\cancel{k}}{\cancel{24}_6} \cdot \frac{\cancel{4}}{(\pi+2)\cancel{k}} = \frac{3\pi+8}{6(\pi+2)}$

$\left(0, \frac{2}{\pi+2}, \frac{3\pi+8}{6(\pi+2)}\right)$

13. $\left(\int_{-\infty}^{\infty} e^{-x^2/2}\,dx\right)^2 = \int_{-\infty}^{\infty} e^{-x^2/2}\,dx \cdot \int_{-\infty}^{\infty} e^{-y^2/2}\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\frac{(x^2+y^2)}{2}}\,dA$

$= \int_0^{2\pi}\int_0^{\infty} e^{-r^2/2}\,r\,dr\,d\theta = \int_{0}^{2\pi} -e^{-r^2/2}\Big|_0^{\infty} d\theta = \int_0^{2\pi} 1\,d\theta = 2\pi$

$\int_{-\infty}^{\infty} e^{-x^2/2}\,dx = \sqrt{2\pi}$

14.
a. $V = 1 \qquad \bar{f} = \frac{1}{1}\int_0^1\int_0^1\int_0^1 z^2+4\,dz\,dy\,dz = \frac{13}{3}$

b. $\frac{3}{4\sqrt{8}\pi}\int_0^{\pi}\int_0^{2\pi}\int_0^{\sqrt{2}} (\rho\cos\theta\sin\varphi + \rho\sin\theta\sin\varphi)\,\rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi = 0$

$V = \frac{4(\sqrt{2})^3\pi}{3}$ (sphere)

b. $V = \int_0^{3}\int_0^{x}\int_0^{9-x^2} dz\,dy\,dx = \frac{81}{4}$

$\frac{4}{81}\int_0^3\int_0^x\int_0^{9-x^2} 3x+2y-z+10\,dz\,dy\,dx = \frac{4}{81}\left(\frac{5427}{20}\right) = \frac{67}{5}$

15. $v(t) = \int t^2\hat{\imath} + (\sin t - t\cos t)\hat{\jmath} + (\cos t + t\sin t)k\,dt =$

$$\left(\tfrac{1}{3}t^3 + C_1\right)\hat{\imath} + \left(-2\cos t - t\sin t + C_2\right)\hat{\jmath} + \left(2\sin t - t\cos t\right)\hat{k} + C_3$$

$C_1 = 1$ $\quad C_2 = 2$ $\quad C_3 = 0$

$$r(t) = \int \left(\tfrac{1}{3}t^3 + 1\right)\hat{\imath} + \left(-2\cos t - t\sin t + 2\right)\hat{\jmath} + \left(2\sin t - t\cos t\right)\hat{k}\,dt$$

$$= \left(\tfrac{1}{12}t^4 + t + C_1\right)\hat{\imath} + \left(t\cos t - 3\sin t + 2t + C_2\right)\hat{\jmath} + \left(-3\cos t - t\sin t\right)\hat{k} + C_3$$

$C_1 = 0$ $\quad C_2 = 1$ $\quad -3 + C_3 = -1$ $\quad C_3 = 2$

$$r(t) = \left(\tfrac{1}{12}t^4 + t\right)\hat{\imath} + \left(t\cos t - 3\sin t + 2t + 1\right)\hat{\jmath} + \left(-3\cos t - t\sin t + 2\right)\hat{k}$$

16. a. $c\int_0^1\int_0^2 x(1+y)\,dy\,dx = \underset{=1}{c(2)} \rightarrow \quad c = \frac{1}{2}$

$P(x \ge \frac{1}{2}) \quad \frac{1}{2}\int_{1/2}^1\int_0^2 x(1+y)\,dy\,dx = \frac{3}{4}$

$P(x \ge \frac{1}{2}, y \le \frac{1}{2}) \quad \frac{1}{2}\int_{1/2}^1\int_0^{1/2} x(1+y)\,dy\,dx = \frac{15}{128}$

$P(x + y \le 1) \quad \frac{1}{2}\int_0^1\int_0^{1-x} x(1+y)\,dy\,dx = \frac{5}{48}$

b. $c\int_0^\infty\int_0^\infty e^{-(\frac{1}{2}x + \frac{1}{5}y)}\,dy\,dx = c(10) = 1 \quad c = \frac{1}{10}$

$P(y \ge 1) = \frac{1}{10}\int_0^\infty\int_1^\infty e^{-(\frac{1}{2}x + \frac{1}{5}y)}\,dy\,dx = e^{-1/15}$

$P(x \le 2, y \le 4) = \frac{1}{10}\int_0^2\int_0^4 e^{-(\frac{1}{2}x + \frac{1}{5}y)}\,dy\,dx = (e-1)(e^{4/5} - 1)e^{-9/5}$