MTH 277, Exam #1, Make-up, Spring 2018 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing" for the answer only; partial credit will not be possible and any credit awarded for the work will not be available.

1. Use $\vec{u} = \langle 3, -1, 6\rangle, \vec{v} = \langle 3, -4, -2\rangle$ to find the angle between $\vec{u}$ and $\vec{v}$. (2 points)

$\vec{u} \cdot \vec{v} = 9 + 4 - 12 = 1$

$\|\vec{u}\| = \sqrt{9+1+36} = \sqrt{46}$

$\|\vec{v}\| = \sqrt{9+16+4} = \sqrt{29}$

$\cos^{-1}\left(\frac{1}{\sqrt{46}\cdot\sqrt{29}}\right) = 1.54$ radians

$88.4°$

2. Find the volume of the parallelepiped bounded by the vectors $\vec{u} = \langle 2,3,-4\rangle, \vec{v} = \langle 1,5,4\rangle$, $\vec{w} = \langle 2,-2,9\rangle$. (2 points)

$$\begin{vmatrix} 2 & 3 & -4 \\ 1 & 5 & 4 \\ 2 & -2 & 9 \end{vmatrix} = 2(5\cdot 9 + 2\cdot 4) - 3(1\cdot 9 - 2\cdot 4) + (-4)(1(-2) - 2\cdot 5)$$

$$= 151$$

$V = 151$

3. State the domain and range of the function $f(x, y) = \frac{\ln(x-y)}{\sqrt{x+y+4}}$ in appropriate notation. (3 points)

$x + y + 4 > 0$

$x + y > -4$

$x - y > 0$

D: $\{(x,y) \mid x+y > -4, x-y > 0\}$

R: $(-\infty, K]$

* this function has a maximum

4. Find the equation of the plane perpendicular to the line $\frac{x-1}{3} = y - 4 = \frac{z-6}{-1}$, passing through the point $(3, -2, 2)$. (3 points)

$\langle 3, 1, -1 \rangle$

$3(x-3) + (y+2) - (z-2) = 0$

5. Identify the quadric surface $z^2 + x^2 - \frac{y^2}{4} = 1$, and convert the equation to cylindrical and spherical coordinates. (5 points)

hyperboloid of one sheet

cylindrical

$z^2 + r^2(\cos^2\theta - \frac{1}{4}\sin^2\theta) = 1$

Spherical

$\rho^2\cos^2\varphi + \rho^2\cos^2\theta\sin^2\varphi - \frac{1}{4}\rho^2\sin^2\theta\sin^2\varphi) = 1$

$$\rho^2 = \frac{1}{\cos^2\varphi + (\cos^2\theta - \frac{1}{4}\sin^2\theta)\sin^2\varphi}$$

6. Write an equation of the cylinder $9x^2 + 16z^2 = 25$ in parametric (surface) form. (3 points)

$\frac{9x^2}{25} + \frac{16z^2}{25} = \frac{25}{25}$

$\frac{x^2}{16} + \frac{z^2}{9} = 1$

$\vec{r}(u,v) = 4\cos u\,\hat{i} + v\,\hat{j} + 3\sin u\,\hat{k}$

7. Consider $\vec{u}(t) = \csc^2 t\,\hat{\imath} + \frac{1}{\sqrt{1-t^2}}\hat{\jmath} + t^{3/2}\hat{k}$. Find: (5 points)

a. $\int \vec{u}(t)\,dt$

$$= (-\cot t + C_1)\hat{\imath} + (\arcsin t + C_2)\hat{\jmath} + (\tfrac{2}{5}t^{5/2} + C_3)\hat{k}$$

b. Describe the continuity of $\vec{u}(t)$.

$t \neq k\pi$

$-1 < t < 1$

$t > 0$

$\boxed{(0,1)}$ only continuous on this interval

8. Find the limits. (3 points each)

a. $\lim\limits_{(x,y)\to(0,0)} \frac{x^2+2xy^2}{x^2+y^2}$

if $y = kx$

$$\lim_{x\to 0} \frac{x^2 + 2x\cdot k^2x^2}{x^2 + k^2x^2} = \lim_{x\to 0} \frac{\cancel{x^2}(1+2xk^2)}{\cancel{x^2}(1+k^2)} =$$

$$\lim_{x\to 0} \frac{1}{1+k^2} + \lim_{x\to 0} \frac{2xk^2}{1+k^2} = \frac{1}{1+k^2} + 0$$

DNE since it depends on k

b. $\lim\limits_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$

$y = kx^2$

$$\lim_{x\to 0} \frac{x^2\cdot kx^2}{x^4 + k^2x^4} = \lim_{x\to 0} \frac{\cancel{x^4}(k)}{\cancel{x^4}(1+k^2)} = \frac{k}{1+k^2}$$

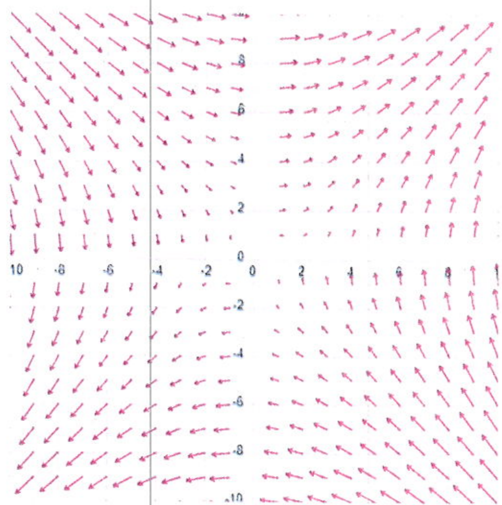
DNE

9. Identify which vector field goes with each graph. (4 points)

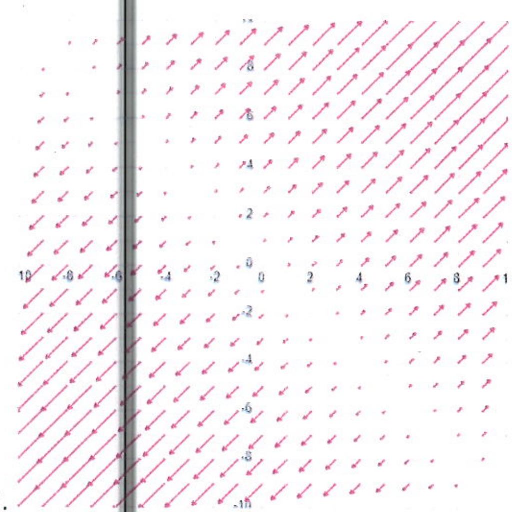
a. $\vec{F}(x,y) = \left(\frac{1}{x} + y\right)\hat{\imath} + \left(x - \frac{1}{y}\right)\hat{\jmath}$

b. $\vec{F}(x,y) = (x+y)\hat{\imath} + (x+y)\hat{\jmath}$

i.



ii.



a

b

10. Find the value of the line integral $\int_C (x + 2\sqrt{y})ds$ on the path between $(-1,3)$ and $(3,6)$. (5 points)

$r(t) = (4t-1)\hat{\imath} + (3t+3)\hat{\jmath} \leftarrow \langle 4,3 \rangle$

$\| \langle 4,3 \rangle \| = 5$

$5\int_0^1 (4t-1) + 2\sqrt{3t+3}\, dt = 5\int_0^1 4t-1 + 2\sqrt{3}(\sqrt{t+1})\,dt =$

$5\left[2t^2 - t + 2\sqrt{3}\cdot\frac{2}{3}(t+1)^{3/2}\right]_0^1 = 5\left[2 - 1 + 2\sqrt{3}\cdot\frac{2}{3}\underbrace{(2)^{3/2}}_{2\sqrt{2}} - 2\sqrt{3}\cdot\frac{2}{3}(1)\right]$

$= 5\left[1 + \frac{8\sqrt{6}}{3} - \frac{4\sqrt{3}}{3}\right] \approx 26.11$

11. Find the total differential of $w = x^2yz^2 + \sin(yz)$. Evaluate it at $w(3,1,\pi)$, and use the total differential to estimate the value of $w(3.1, 0.95, 3)$. (3 points)

$\Delta w = (2xyz^2)\Delta x + (x^2z^2 + z\cos(yz))\Delta y + (2x^2yz + y\cos yz)\Delta z$

$2\cdot 3\cdot 1\cdot \pi^2 \qquad 9\cdot\pi^2 + \pi\cos\pi \qquad 2\cdot 9\cdot 1\cdot\pi + 1\cdot\cos\pi$

$6\pi^2\Delta x + (9\pi^2 - \pi)\Delta y + (18\pi - 1)\Delta z$

$= 6\pi^2(.1) + (9\pi^2 - \pi)(-.05) + (18\pi - 1)(-.14)$

≈ -6.22776

$w(3.1, 0.95, 3) \approx 3^2(1)(\pi)^2 + \sin\pi + \Delta w = 9\pi^2 + \Delta w \approx 82.599$