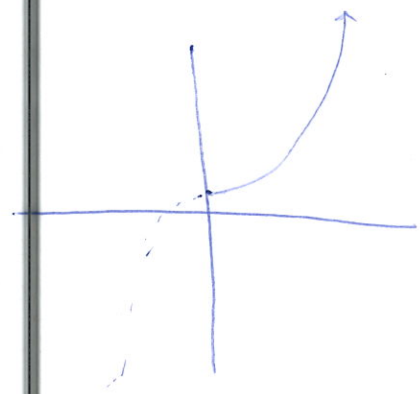
MTH 174, Quiz #9, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Sketch the parametric curves. Rewrite each in rectangular coordinates. Be sure to label the orientation of the curve.

   a. $x = e^t, y = e^{3t} + 1$

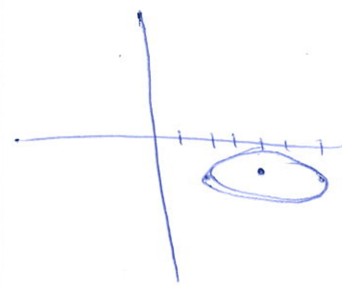
   $y = t^3 + 1$



   b. $x = 4 + 2\cos\theta, y = -1 + \sin\theta$

   $(x-4) = 2\cos\theta$ $y+1 = \sin\theta$

   $\frac{x-4}{2} = \cos\theta$



   $\frac{(x-4)^2}{4} + (y+1)^2 = 1$

2. Find the slope of the tangent line and the concavity at $\theta = \frac{\pi}{6}$ for the parametric curve $x = 2 + \sec\theta, y = 1 + 2\tan\theta$. Are there any points of vertical tangency?

$x' = \sec\theta\tan\theta$

$y' = 2\sec^2\theta$

$\frac{dy}{dx} = \frac{y'}{x'} = \frac{2\sec^2\theta}{\sec\theta\tan\theta} = 2\sec\theta\cdot\cot\theta$

$= 2\cdot\frac{1}{\cancel{\cos\theta}}\cdot\frac{\cancel{\cos\theta}}{\sin\theta} = 2\csc\theta$

$\frac{dy}{dx}$ = horizontal $= 0$ at no values of θ

$\frac{dy}{dx} = 2\csc(\pi/6) = 4$

$\frac{dy}{dx}$ = vertical = undefined at $0, \pm\pi, \pm 2\pi$, etc.

$\frac{d^2y}{dx^2} = \frac{(y')'}{x'} = \frac{-2\csc\theta\cot\theta}{\sec\theta\tan\theta} = -2\frac{1}{\sin\theta}\left(\frac{\cos\theta}{\sin\theta}\right)\frac{1}{\cos\theta}^{2}$

$= \frac{-2\cos^3\theta}{\sin^3\theta} = -2\cot^3\theta = -\frac{2}{3\sqrt{3}}$ down