MTH 174, Quiz #4, Spring 2018 Name KEY

**Instructions**: Show all work. Use exact answers unless otherwise asked to round.

1. Rewrite $\int \sec^4 4x \tan 4x \, dx$ so that it can be integrated with substitution. You do not need to integrate.

a) $\int \sec^3 4x(\sec 4x \tan 4x)\, dx$

$u = \sec 4x$

$du = 4\sec 4x \tan 4x\, dx$

$\frac{1}{4}\int u^3\, du$

b) $\int \sec^2 4x\,(\sec^2 4x) \tan 4x\, dx$

$u = \tan 4x$

$du = 4\sec^2 4x\, dx$

$\int \sec^2 4x\,(1+\tan^2 4x)\tan 4x\, dx$

$\frac{1}{4}\int (1+u^2)u\, du$

2. Use trigonometric substitution to integrate $\int \frac{\sqrt{25x^2+4}}{x^4}\, dx$.

$\frac{2}{5}\tan\theta = x$

$\frac{4}{25}\tan^2\theta = x^2$

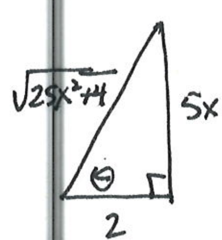
$25x^2+4 = 4\tan^2\theta + 4 = 4(\tan^2\theta + 1) = 4\sec^2\theta$

$\sqrt{25x^2+4} = 2\sec\theta$

$dx = \frac{2}{5}\sec^2\theta\, d\theta$

$x^4 = \left(\frac{2}{5}\tan\theta\right)^4 = \frac{16}{625}\tan^4\theta$

$\tan\theta = \frac{x\cdot 5}{2}$




$= \int \frac{2\sec\theta\cdot\frac{2}{5}\sec^2\theta\, d\theta}{\left(\frac{16}{625}\right)\tan^4\theta} =$

$\int \frac{\cancel{4}}{\cancel{5}}\cdot\frac{\overset{125}{\cancel{625}}}{\underset{4}{\cancel{16}}}\cdot\frac{1}{\cancel{\cos^3\theta}}\cdot\frac{\cos^{\cancel{4}}\theta}{\sin^4\theta}\, d\theta$

$\int \frac{125}{4}\cos\theta\cdot(\sin\theta)^{-4}\, d\theta$

$u = \sin\theta$

$du = \cos\theta$

$\frac{125}{4}\int u^{-4}\, du$

$\frac{125}{4}\cdot\frac{u^{-3}}{-3} + C = \frac{-125}{12}\csc^3\theta + C$

$= \boxed{\frac{-125}{12}\left(\frac{\sqrt{25x^2+4}}{5x}\right)^3 + C}$