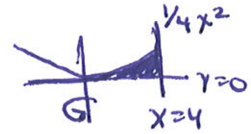
MTH 174, Quiz #19, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the volume of the solid rotated around the y-axis and is bounded by $y = \frac{1}{4}x^2, y = 0, x = 4$.



$$V = 2\pi\int_0^4 x\left(\tfrac{1}{4}x^2\right)dx = \frac{2\pi}{4}\int_0^4 x^3\,dx =$$

$$\frac{\pi}{2}\cdot\frac{x^4}{4}\Big|_0^4 = \frac{\pi}{8}(256) = 32\pi$$

2. Find the area of the surface of revolution bounded by $y = \sqrt{4 - x^2}$ on $[-1,1]$ around the x-axis.

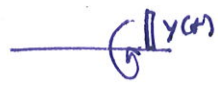
$$y' = \frac{-x}{\sqrt{4-x^2}}$$

$$S = 2\pi\int_{-1}^{1}\sqrt{4-x^2}\cdot\sqrt{1+\frac{x^2}{4-x^2}}\,dx =$$

$$2\pi\int_{-1}^{1}\cancel{\sqrt{4-x^2}}\cdot\sqrt{\frac{4-x^2+x^2}{\cancel{4-x^2}}}\,dx = 2\pi\int_{-1}^{1}2\,dx = 4\pi(2) = 8\pi$$

3. Set up an integral to calculate the area of the surface of revolution bounded by the parametric curve $x = t + \sin t, y = t + \cos t$ on $\left[0, \frac{\pi}{2}\right]$ revolved around the x-axis. Do not integrate.

$x' = 1 + \cos t \quad y' = 1 - \sin t$



$$2\pi\int_0^{\pi/2}(t+\cos t)\sqrt{(1+\cos t)^2+(1-\sin t)^2}\,dt = 2\pi\int_0^{\pi/2}(t+\cos t)\sqrt{1+2\cos t+\cos^2 t +1-2\sin t+\sin^2 t}\,dt$$

$$= 2\pi\int_0^{\pi/2}(t+\cos t)\sqrt{3+2\cos t-2\sin t}\,dt$$