

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the volume of the solid rotated around the x-axis and is bounded by $y = 2$, $y = 4 - \frac{x^2}{4}$.

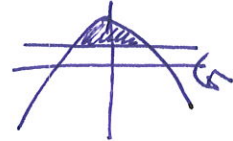
$$V = \pi \int_{-2\sqrt{2}}^{2\sqrt{2}} \left(4 - \frac{x^2}{4}\right)^2 - (2)^2 dx$$

$$\pi \int_{-2\sqrt{2}}^{2\sqrt{2}} 16 - 2x^2 + \frac{x^4}{16} - 4 dx =$$

$$\pi \int_{-2\sqrt{2}}^{2\sqrt{2}} 12 - 2x^2 + \frac{x^4}{16} dx = 2\pi \int_0^{2\sqrt{2}} 12 - 2x^2 + \frac{x^4}{16} dx$$

$$= 2\pi \left[12x - \frac{2}{3}x^3 + \frac{x^5}{80} \right]_0^{2\sqrt{2}} = 2\pi \left[24\sqrt{2} - \frac{2}{3} \cdot 16\sqrt{2} + \frac{128}{80}\sqrt{2} \right] = \frac{448\pi\sqrt{2}}{15}$$

$$\approx 132.69$$



$$2 = 4 - \frac{x^2}{4}$$

$$\frac{x^2}{4} = 2$$

$$x^2 = 8$$

$$x = \pm 2\sqrt{2}$$

2. Find the volume of the solid rotated around the line $x = 3$, bounded by $y = \sqrt{x}$, $y = 0$, $x = 3$.

$$V = 2\pi \int_0^3 (3-x)\sqrt{x} dx =$$

$$2\pi \int_0^3 3\sqrt{x} - x^{3/2} dx =$$

$$2\pi \left[3 \cdot \frac{2}{3} x^{3/2} - \frac{2}{5} x^{5/2} \right]_0^3 = 2\pi \left[2 \cdot 3\sqrt{3} - \frac{2}{5} \cdot 9\sqrt{3} \right] =$$

$$\frac{24\sqrt{3}\pi}{5} \approx 26.12$$

