MTH 174, Quiz #17, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Find the Maclaurin series for $f(x) = xe^{-x}, n = 4$.

$$f(x) = x\sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = x\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n!}$$

$$f(x) \approx \frac{(-1)^0 x^1}{0!} + \frac{(-1)^1 x^2}{1!} + \frac{(-1)^2 x^3}{2!} + \frac{(-1)^3 x^4}{3!} + \frac{(-1)^4 x^5}{4!}$$

$$= x - x^2 + \frac{1}{2}x^3 - \frac{1}{6}x^4 + \frac{1}{24}x^5$$

2. Use the Taylor/power series in the table that follows to evaluate and find a power series for the following:

a. $f(x) = e^x + e^{-x} = 2\cosh x$

$$\sum_{n=0}^{\infty} \frac{x^n}{n!} + \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} = \sum_{n=0}^{\infty} \frac{2x^{2n}}{(2n)!}$$

$1 + \cancel{x} + \frac{x^2}{2} + \cancel{\frac{x^3}{6}} + \cdots + 1 - \cancel{x} + \frac{x^2}{2} - \cancel{\frac{x^3}{6}} \ldots$

odd terms cancel
even terms survive

b. $f(x) = \int_0^x e^{-t^2} - 1\,dt$ $= \int_0^x \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{n!} - 1\,dt = \sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{n!\,(2n+1)} - t \Big|_0^x =$

$$\sum_{n=\cancel{0}\,1}^{\infty} \frac{(-1)^n x^{2n+1}}{n!\,(2n+1)} - \cancel{x} = \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n+1}}{n!\,(2n+1)}$$

$n=0$ $\frac{x}{1}$

c. $g(x) = \frac{1}{2i}\left(e^{ix} - e^{-ix}\right) = \sin x$

$$\frac{1}{2i}\left[\sum_{n=0}^{\infty} \frac{(i)^n x^n}{n!} - \frac{(-i)^n x^n}{n!}\right] = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$n=0$ $1-1=0$ $n=3$ $-i-i=-2i$
$n=1$ $i+i=2i$
$n=2$ $-1+1=0$

2i's cancel w/ $\frac{1}{2i}$
odd terms survive alternating sign

3. Find the maximum error on the Taylor polynomial for $\frac{1}{x^2}$ centered at $c = 2, n = 4$ at $x = 2.1$.

n	$n!$	$f^{(n)}(x)$	$f^{(n)}(2)$	$(x-2)^n$
0	$0! = 1$	x^{-2}	$\frac{1}{4}$	1
1	$1! = 1$	$-2x^{-3}$	$-1/4$	$(x-2)$
2	$2! = 2$	$6x^{-4}$	$3/8$	$(x-2)^2$
3	$3! = 6$	$-24x^{-5}$	$-3/4$	$(x-2)^3$
4	$4! = 24$	$120x^{-6}$	$15/8$	$(x-2)^4$
5	$5! = 120$	$-720x^{-7}$	$-45/8$	$(x-2)^5$

-5.625

@ 2.1 -3.997

$$\left| \frac{-45}{8} \cdot \frac{1}{120} (0.1)^5 \right| < -4.6875 \times 10^{-7}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots \qquad R = 1$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \qquad R = \infty$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \qquad R = \infty$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \qquad R = \infty$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \qquad R = 1$$

$$\ln(1+x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \qquad R = 1$$

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} x^2 + \frac{k(k-1)(k-2)}{3!} x^3 + \dots \qquad R = 1$$