MTH 174, Quiz #14, Spring 2018 Name KEY

**Instructions**: Show all work. Use exact answers unless otherwise asked to round.

1. Find the number of terms needed to estimate the sum $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n^3-1}$ within 0.0001 of the true sum.

$$\frac{1}{2n^3-1} \le .0001 \Rightarrow 2n^3-1 \ge 10^4$$

$$n^3 \ge 4999.5$$

$$n \ge 17.099$$

$$\boxed{n = 18}$$

2. Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ converges absolutely or conditionally.

$$\left| \frac{(-1)^n}{n \ln n} \right| = \frac{1}{n \ln n} \qquad \int_2^{\infty} \frac{1}{x \ln x}\, dx = \ln(\ln x)\Big|_2^{\infty}$$ diverges by integral test

$$\lim_{n\to\infty} \frac{1}{n \ln n} \to 0$$ Converges conditionally by alternating series test

3. Determine whether the series converges or diverges.

a. $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{4n}}{(2n+1)!}$

$$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+1}}\, 2^{\cancel{4n}+4}}{(2n+3)!} \cdot \frac{(2n+1)!}{\cancel{(-1)^n}\, \cancel{2^{4n}}} \right| = \lim_{n\to\infty} \left| \frac{2^4}{(2n+2)(2n+3)} \right| = 0 < 1$$

Converges by ratio test

b. $\sum_{n=1}^{\infty} \frac{5^n}{n^4}$

$$\lim_{n\to\infty} \left| \frac{5^{\cancel{n}+1}}{(n+1)^4} \cdot \frac{n^4}{\cancel{5^n}} \right| = \lim_{n\to\infty} |5| \cdot \left| \cancel{\lim_{n\to\infty} \frac{n^4}{(n+1)^4}} \to 1 \right| = 5 > 1$$

diverges by ratio test

c. $\sum_{n=1}^{\infty}\left(-\frac{3n}{2n+1}\right)^{3n}$

$$\lim_{n\to\infty} \sqrt[n]{\left|\left(-\frac{3n}{2n+1}\right)^{3n}\right|} = \lim_{n\to\infty} \left|\frac{3n}{2n+1}\right|^3 = \left(\frac{3}{2}\right)^3 = \frac{27}{8} > 1$$

diverges by the root test

d. $\sum_{n=1}^{\infty}\frac{n5^n}{n!}$

$$\lim_{n\to\infty} \left|\frac{(n+1)\,5^{\cancel{n}+1}}{(n+1)!} \cdot \frac{n!}{n\,\cancel{5^n}}\right| = \lim_{n\to\infty} \left|\cancel{\frac{n+1}{n}}\right| \cdot |5| \cdot \left|\cancel{\frac{1}{n+1}}\right| = 0 < 1$$

Converges by ratio test