MTH 174, Quiz #13, Spring 2018 Name KEY

Instructions: Show all work. Use exact answers unless otherwise asked to round.

1. Use the integral test to determine whether $\sum_{n=1}^{\infty} \frac{\arctan n}{n^2+1}$ converges or diverges.

$\int_1^{\infty} \frac{\arctan x}{x^2+1}\,dx$ $u = \arctan x$

$du = \frac{1}{x^2+1}\,dx$

$\int_{\pi/4}^{\pi/2} u\,du = \frac{1}{2}u^2 \Big|_{\pi/4}^{\pi/2} = \frac{1}{2}\left[\frac{\pi^2}{2^2} - \frac{\pi^2}{4^2}\right] =$

$\frac{1}{2}\left[\frac{\pi^2}{4} - \frac{\pi^2}{16}\right] = \frac{3\pi^2}{32}$ converges

2. Determine if the series converges.

a. $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$ by integral test

$\int_2^{\infty} \frac{1}{x\sqrt{x^2-1}}\,dx = \operatorname{arcsec} x \Big|_2^{\infty} = \lim_{b\to\infty} \operatorname{arcsec} b - \operatorname{arcsec} 2$

$\pi/2 - \pi/3 = \pi/6$

converges

b. $\sum_{n=1}^{\infty} \frac{2^n+1}{5^n+1}$ compare w/ $\left(\frac{2}{5}\right)^n$ converges by ratio test

$\lim_{n\to\infty} \frac{\frac{2^n+1}{5^n+1}}{\frac{2^n}{5^n}} = \lim_{n\to\infty} \frac{(2^n+1)5^n}{(5^n+1)2^n} = 1$ converges also

c. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1}$ direct comparison w/ $\frac{1}{\sqrt{n}} < \frac{1}{\sqrt{n}-1}$

$\sum \frac{1}{\sqrt{n}}$ diverges by p-series

$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n}-1}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}-1} = 1$ diverges also

3. Find the value of N needed to estimate the sum of $\sum_{n=0}^{\infty} e^{-n/2}$ with an error of less than 0.001.

$\int_N^{\infty} e^{-n/2}\,dn = -2e^{-n/2}\Big|_N^{\infty} = -2(0) + 2e^{-N/2} \leq .001$

$\ln\left(e^{-N/2} = .0005\right)$

$-N/2 = -7.6\ldots$

$N/2 = 7.6\ldots$

$N = 15.2\ldots$

$\boxed{N = 16}$