174 Homework #4 Key

1a. $\int_0^{\pi/2} \cos 5x \cos 2x \, dx$ $\quad a=5, b=2$

(4.19) $\int \cos(ax)\cos(bx)\,dx = \frac{1}{2}\left[\frac{\sin(a+b)x}{a+b} + \frac{\sin(a-b)x}{a-b}\right] + C$

$\Rightarrow \frac{1}{2}\left[\frac{\sin 7x}{7} + \frac{\sin 3x}{3}\right]_0^{\pi/2} = \frac{1}{2}\left[\frac{1}{7}(-1) + \frac{1}{3}(-1)\right] = -\frac{5}{21}$

b. $\int_0^2 x^2\sqrt{4-x^2}\,dx$ $\quad a=2$

(3.30) $\int x^2\sqrt{a^2-x^2}\,dx = \frac{x}{8}(2x^2-a^2)\sqrt{a^2-x^2} + \frac{a^4}{8}\arcsin\left(\frac{x}{a}\right) + C$

$\Rightarrow \frac{x}{8}(2x^2-4)\sqrt{4-x^2} + \frac{16}{8}\arcsin\left(\frac{x}{2}\right)\Big|_0^2 =$

$\frac{1}{4}(8-4)(0) + 4\arcsin(1) - (0) - 4\arcsin(0) = 2\pi$

c. $\int \frac{\tan^3\left(\frac{1}{z}\right)}{z^2}\,dz$ $\quad u = \frac{1}{z}$, $du = \frac{-1}{z^2}dz$ $\quad -\int \tan^3 u\,du$

(4.35) $\int \tan^3(ax)\,dx = \frac{1}{a}\ln|\cos(ax)| + \frac{1}{2a}\sec^2(ax) + C$

$\Rightarrow -\ln\left|\cos\frac{1}{z}\right| + -\frac{1}{2}\sec^2\left(\frac{1}{z}\right) + C$

d. $\int \frac{\sin 2\theta}{\sqrt{5-\sin\theta}}\,d\theta = \int \frac{2\sin\theta\cos\theta}{\sqrt{5-\sin\theta}}\,d\theta$ $\quad u=\sin\theta$, $du = \cos\theta\,d\theta$

$= \int \frac{2u\,du}{\sqrt{5-u}}$ $\quad a=-1,\ b=5$

(3.7) $\int \frac{x}{\sqrt{ax+b}}\,dx = \frac{2x}{a}\sqrt{ax+b} - \frac{4b}{3a^2}(ax+b)^{3/2} + C$

$\Rightarrow 2\left[\frac{2\sin\theta}{-1}\sqrt{5-\sin\theta} - \frac{20}{3}(5-\sin\theta)^{3/2}\right] + C$

e. $\int \sin^2 x\cos x\ln(\sin x)\,dx$ $\quad u=\sin x \quad du = \cos x\,dx$

$\int u^2 \ln u\,du$

(7.3) $\int x^n \ln x\,du = -\frac{1}{(n+1)^2}x^{n+1} + \frac{1}{n+1}x^{n+1}\ln x + C$

$-\frac{1}{9}x^3 + \frac{1}{3}x^3\ln x + C$

1f. $\int \sqrt{e^{2t}-1}\, dt = \int \sqrt{e^{2t}-1} \cdot \frac{e^t}{e^t}\, dt =$ $\quad \frac{u = e^t}{du = e^t dt}$ $\quad \int \frac{\sqrt{u^2-1}}{u}\, du$

(3.32) $\int \frac{\sqrt{x^2-a^2}}{x}\, dx = \sqrt{x^2-a^2} - a \operatorname{arcsec}\left|\frac{x}{a}\right| + C$ $\quad a = 1$,

$\Rightarrow \sqrt{e^{2t}-1} - \operatorname{arcsec}(e^t) + C$

g. $\int e^t \sin(\alpha t - 3)\, dt$ $\quad u = \alpha t - 3 \quad \rightarrow \frac{u+3}{\alpha} = t = \frac{u}{\alpha} + \frac{3}{\alpha}$

$du = \alpha\, dt \rightarrow \frac{1}{\alpha} du = dt$

$\int e^{u/\alpha} e^{3/\alpha} \sin u \frac{1}{\alpha} du$ $\quad a = \frac{1}{\alpha},\ b = 1$

(8.11) $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}\left[a \sin bx - b \cos bx\right] + C$

$\Rightarrow e^{3/\alpha} \frac{e^{\frac{1}{\alpha}(\alpha t - 3)}}{\frac{1}{\alpha^2}+1} \frac{\alpha^2}{\alpha^2}\left[\frac{1}{\alpha} \sin(\alpha t - 3) - \cos(\alpha t - 3)\right] + C$

$= \frac{e^t \alpha^2}{\alpha^2+1}\left[\frac{1}{\alpha} \sin(\alpha t - 3) - \cos(\alpha t - 3)\right] + C$

h. $\int \frac{5t}{(2t+1)^7}\, dt$ $\quad a = 2,\ b = 1,\ n = 7$

(2.10) $\int \frac{x}{(ax+b)^n}\, dx = \frac{1}{a^2}\left[\frac{b}{(n-1)(ax+b)^{n-1}} - \frac{1}{(n-2)(ax+b)^{n-2}}\right] + C$

$\Rightarrow \frac{5}{4}\left[\frac{1}{6(2x+1)^6} - \frac{1}{5(2x+1)^5}\right] + C$

i. $\int \sqrt{\frac{x-7}{x+7}}\, dx$ $\quad a = 7$

(3.16) $\int \sqrt{\frac{x-a}{x+a}}\, dx = \sqrt{x^2-a^2} - a \ln\left|\sqrt{x^2-a^2} + x\right| + C$

$\Rightarrow \sqrt{x^2-49} - 7 \ln\left|\sqrt{x^2-49} + x\right| + C$

j. $\int \frac{\sqrt{16 + \frac{1}{x^2}}}{x^3}\, dx$ $\quad u = \frac{1}{x^2} = x^{-2}$ $\quad -\frac{1}{2}\int \sqrt{16+u}\, du$

$du = -2x^{-3} \rightarrow -\frac{1}{2} du = \frac{1}{x^3}$ $\quad a = 1,\ b = 16$

(3.1) $\int \sqrt{ax+b}\, dx = \frac{2}{3a}(ax+b)^{3/2} + C$

$\rightarrow -\frac{1}{2}\left[\frac{2}{3}\left(16 + \frac{1}{x^2}\right)^{3/2}\right] + C = -\frac{1}{3}\left(16 + \frac{1}{x^2}\right)^{3/2} + C$

1k. $\int \sin^2 2\varphi \cos 3\varphi \, d\varphi$ $\quad a=2,\ b=3$

(4.20) $\int \sin^2(ax)\cos(bx)\,dx = -\frac{1}{4(2a-b)}\sin[(2a-b)x] + \frac{1}{2b}\sin bx - \frac{1}{4(2a+b)}\sin[(2a+b)x] + C$

$\Rightarrow \frac{-1}{4(4-3)}\sin x - \frac{1}{6}\sin 3x - \frac{1}{4(4+3)}\sin(7x) + C$

$= -\frac{1}{4}\sin x - \frac{1}{6}\sin 3x - \frac{1}{28}\sin 7x + C$

l. $\int x^4 \arctan x\,dx =$ $\quad n=4$

(5.8) $\int x^n \arctan x\,dx = \frac{1}{n+1}x^{n+1}\arctan x - \frac{1}{n+1}\int \frac{x^{n+1}}{x^2+1}\,dx + C$

$\frac{1}{5}x^5\arctan x - \frac{1}{5}\int \frac{x^5}{x^2+1}\,dx + C$ $\quad n=5$

(2.28) $\int \frac{x^n}{x^2 \pm 1}\,dx = \frac{1}{n-1}x^{n-1} \mp \int \frac{x^{n-2}}{x^2 \pm 1}\,dx + C$

$\frac{1}{5}x^5\arctan x - \frac{1}{5}\left[\frac{1}{4}x^4 - \int \frac{x^3}{x^2+1}\,dx\right] + C$ $\quad n=3$

(2.14)

$\frac{1}{5}x^5\arctan x - \frac{1}{20}x^4 + \frac{1}{5}\left[\frac{1}{2}x^2 - \int \frac{x}{x^2+1}\,dx\right] + C$

$\frac{1}{5}x^5\arctan x - \frac{1}{20}x^4 + \frac{1}{10}x^2 - \frac{1}{10}\ln|x^2+1| + C$

m. $\int q^2 \ln(q^6+9)\,dq$ $\quad u=q^3$, $du=3q^2dq$, $\frac{1}{3}du = q^2dq$ $\quad \frac{1}{3}\int \ln(u^2+9)\,du$ $\quad a=3$

(7.5) $\int \ln(x^2+a^2)\,dx = x\ln(x^2+a^2) + 2a\arctan\left(\frac{x}{a}\right) - 2x + C$

$\Rightarrow \frac{1}{3}x\ln(q^6+9) + \frac{2(3)}{3}\arctan\left(\frac{q^3}{3}\right) - \frac{1}{3}2(q^3) + C$

$= \frac{1}{3}x\ln(q^6+9) + 2\arctan\left(\frac{q^3}{3}\right) - \frac{2}{3}q^3 + C$

n. $\int \log_3(4x-7)\,dx$ $\quad u=4x-7$, $du=4dx$, $\frac{1}{4}du = dx$ $\quad \frac{1}{4}\int \log_3 u\,du$ $\quad a=3\quad b=1$

(7.14) $\int \log_a bx\,dx = \frac{1}{\ln a}[x\ln bx - x] + C$

$\Rightarrow \frac{1}{4}\cdot\frac{1}{\ln 3}[\cancel{u}\ln(4x-7) - u] + C \Rightarrow \frac{1}{4\ln 3}[(4x-7)\ln(4x-7) - (4x-7)] + C$

10. $\int y e^{3y} \sin 3y \, dy$ $\quad u = 3y \quad y = \frac{1}{3}u \quad du = 3dy \quad \frac{1}{3}du = dy \quad \frac{1}{9}\int u e^u \sin u \, du$

(8.13) $\int x e^x \sin x \, dx = \frac{1}{2} e^x [\cos x - x\cos x + x \sin x] + C$

$\frac{1}{9}\left[\frac{1}{2} e^{3y}(\cos 3y - 3y\cos 3y + 3y \sin 3y)\right] + C$

$\frac{1}{18} e^{3y} \cos 3y - \frac{1}{6} e^{3y} y \cos 3y + \frac{1}{6} y e^{3y} \sin 3y + C$

p. $\int_1^2 \sqrt{4x^2 - 3}\, dx$ $\quad u = 2x \quad du = 2dx \quad \frac{1}{2}du = dx \quad \frac{1}{2}\int \sqrt{u^2 - 3} \quad a = \sqrt{3}$

(3.26) $\int \sqrt{x^2 \pm a^2}\, dx = \frac{1}{2} x \sqrt{x^2 \pm a^2} \pm \frac{1}{2} a^2 \ln |x + \sqrt{x^2 \pm a^2}| + C$

$\frac{1}{2}\left[\frac{1}{2}\left(2x\sqrt{4x^2-3} - \frac{1}{2}(3) \ln |2x + \sqrt{4x^2-3}|\right)\right] + C$

$\frac{1}{2} x \sqrt{4x^2 - 3} - \frac{3}{8} \ln |2x + \sqrt{4x^2-3}| \Big|_1^2$

$\frac{1}{2}(2)\sqrt{8-3} - \frac{3}{8}\ln|4 + \sqrt{5}| - \frac{1}{2}\sqrt{4-3} + \frac{3}{8}\ln|2 + \sqrt{1}|$

$\sqrt{5} - \frac{3}{8}\ln|4+\sqrt{5}| - \frac{1}{2} + \frac{3}{8}\ln 3$

q. $\int \frac{\cos x}{\sin^2 x - 9}\, dx$ $\quad u = \sin x \quad du = \cos x \, dx \quad \int \frac{du}{u^2 - 9} \quad a = 3$

(2.26) $\int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \ln \left|\frac{x-a}{x+a}\right| + C$

$\frac{1}{6} \ln \left|\frac{\sin x - 3}{\sin x + 3}\right| + C$

r. $\int x \sin x^2 \cos(3x^2)\, dx$ $\quad u = x^2 \quad du = 2x dx \quad \frac{1}{2}du = x dx \quad \frac{1}{2}\int \sin u \cos 3u \, du \quad a = 3, b = 1$

(4.17) $\int \cos(ax)\sin(bx)\, dx = \frac{1}{2(a-b)}\cos[(a-b)x] - \frac{1}{2(a+b)}\cos[(a+b)x] + C$

$\Rightarrow \frac{1}{2}\left[\frac{1}{2(3-1)}\cos 2x - \frac{1}{2(3+1)}\cos 4x + C\right] = \frac{1}{8}\cos 2x - \frac{1}{16}\cos 4x + C$

s. $\int \sin^6 2x \, dx$ $\quad u = 2x \quad du = 2dx \quad \frac{1}{2}du = dx$

(4.5) $\int \sin^n x\, dx = \frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x \, dx$

$\Rightarrow \frac{1}{2}\left[\frac{1}{6}\sin^5 2x \cos 2x + \frac{5}{6}\int \sin^4 2x \, dx\right] + C$

1s cont'd

$\frac{1}{12}\sin^5 2x\cos 2x + \frac{5}{12}\int \sin^4 2x\,dx + C \quad a=2$

(4.4) $\int \sin^4 ax\,dx = -\frac{1}{4a}\sin^3(ax)\cos(ax) - \frac{3}{8a}\sin(ax)\cos(ax) + \frac{3}{8}x + C$

$\frac{1}{12}\sin^5 2x\cos 2x + \frac{5}{12}\left[\frac{1}{8}\sin^3 2x\cos 2x - \frac{3}{16}\sin 2x\cos 2x + \frac{3}{8}x\right] + C$

$\frac{1}{12}\sin^5 2x\cos 2x + \frac{5}{96}\sin^3 2x\cos 2x - \frac{15}{192}\sin 2x\cos 2x + \frac{15}{96}x + C$

t. $\int \sec^5 x\,dx$

(4.41) $\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2}x\tan x + \frac{n-2}{n-1}\int \sec^{n-2}x\,dx$

$\Rightarrow \frac{1}{4}\sec^3 x\tan x + \frac{3}{4}\int \sec^3 x\,dx$

(4.40) $\int \sec^3 x\,dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$

$\Rightarrow \frac{1}{4}\sec^3 x\tan x + \frac{3}{4}\left[\frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x|\right] + C$

$\frac{1}{4}\sec^3 x\tan x + \frac{3}{8}\sec x\tan x + \frac{3}{8}\ln|\sec x + \tan x| + C$

u. $\int \frac{1}{\sqrt{1+\sqrt[3]{x}}}\,dx$

$u = \sqrt{1+\sqrt[3]{x}}$

$u^2 = 1+\sqrt[3]{x}$

$u^2 - 1 = \sqrt[3]{x}$

$(u^2-1)^3 = x$

$3(u^2-1)^2\, 2u\,du = dx$

$\int \frac{3(u^2-1)^2\, 2\cancel{u}\,du}{\cancel{u}}$

$6\int (u^2-1)^2\,du = 6\int u^4 - 2u^2 + 1\,du$

$= 6\left[\frac{u^5}{5} - \frac{2}{3}u^3 + u\right] + C$

$6\left[\frac{1}{5}(1+\sqrt[3]{x})^{5/2} - \frac{2}{3}(1+\sqrt[3]{x})^{3/2} + (1+\sqrt[3]{x})^{1/2}\right] + C$

$\frac{6}{5}(1+\sqrt[3]{x})^{5/2} - 4(1+\sqrt[3]{x})^{3/2} + 6(1+\sqrt[3]{x})^{1/2} + C$

v. $\int \frac{1}{2x^2-3x+5}\,dx \qquad a=2,\ b=-3,\ c=5 \qquad 9 < 4(2)(5)$

$40 - 9 = 31$

(2.23) $\int \frac{1}{ax^2+bx+c}\,dx = \frac{2}{\sqrt{4ac-b^2}}\arctan\left(\frac{2ax+b}{\sqrt{4ac-b^2}}\right) + C$

$\Rightarrow \frac{2}{\sqrt{31}}\arctan\left(\frac{4x-3}{\sqrt{31}}\right) + C$

w. $\int \frac{\sqrt{e^x+1}}{e^{3x}}\,dx \cdot \frac{e^x}{e^x} \qquad u = e^x \qquad du = e^x\,dx \qquad \int \frac{\sqrt{u+1}}{u^4}\,du$

(3.13) $\int \frac{\sqrt{ax+b}}{x^n}\,dx = \frac{1}{b(1-n)}\left[\frac{(ax+b)^{3/2}}{x^{n-1}} + \frac{(2n-5)a}{2}\int \frac{1}{x^{n-1}\sqrt{ax+b}}\,dx\right] + C$

1w cont'd $a=1,\ b=1,\ n=4$ $u=e^x$

$$-\frac{1}{3}\left[\frac{(u+1)^{3/2}}{u^3} + \frac{3}{2}\int \frac{1}{u^3\sqrt{u+1}}\,du\right]+C$$

$n=3$

(3.11) $$\int \frac{1}{x^n\sqrt{ax+b}}\,dx = \frac{1}{a(1-n)}\left[\frac{\sqrt{ax+b}}{x^{n-1}} + \frac{(2n-3)b}{2}\int\frac{1}{x^{n-1}\sqrt{ax+b}}\,dx\right]$$

$$-\frac{1}{3}\left[\frac{(u+1)^{3/2}}{u^3} + \frac{3}{2}\left[\frac{1}{-2}\left(\frac{\sqrt{u+1}}{u^2} + \frac{3}{2}\int\frac{1}{u^2\sqrt{u+1}}\,du\right)\right]\right]+C$$

$n=2$

$$-\frac{(u+1)^{3/2}}{3u^3} + \frac{1}{4}\frac{\sqrt{u+1}}{u^2} - \frac{3}{8}\int\frac{1}{u^2\sqrt{u+1}}\,du = -\frac{(u+1)^{3/2}}{3u^3} + \frac{\sqrt{u+1}}{4u^2} - \frac{3}{8}\left[\frac{\sqrt{u+1}}{u} + \frac{1}{2}\int\frac{1}{u\sqrt{u+1}}\,du\right.$$

$$= -\frac{(u+1)^{3/2}}{3u^3} + \frac{\sqrt{u+1}}{4u^2} - \frac{3\sqrt{u+1}}{8u} - \frac{3}{16}\ln\left|\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}\right|+C$$

(3.10) $$\int\frac{1}{x\sqrt{ax+b}}\,dx = \frac{1}{\sqrt{b}}\ln\left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}-\sqrt{b}}\right|+C$$

$$\Rightarrow -\frac{(e^x+1)^{3/2}}{3e^{3x}} + \frac{\sqrt{e^x+1}}{4e^{2x}} - \frac{3\sqrt{e^x+1}}{8e^x} - \frac{3}{16}\ln\left|\frac{\sqrt{e^x+1}-1}{\sqrt{e^x+1}+1}\right|+C$$

X. $$\int \sin\theta\cos\theta\sqrt{\cos^2\theta+5\cos\theta+4}\,d\theta$$

$u=\cos\theta$
$du=-\sin\theta\,d\theta$

$$-\int u\sqrt{u^2+5u+4}\,du$$

$a=1,\ b=5,\ c=4$

(3.21) $$\int x\sqrt{ax^2+bx+c}\,dx = \frac{\sqrt{ax^2+bx+c}}{24a^2} - \frac{4ax^2+bx+4c}{24a^{3/2}} + \frac{b(b^2-4ac)}{16a^{3/2}}\cdot \ln\left|b+2ax+2\sqrt{a(ax^2+bx+c)}\right|+C$$

$$-\frac{\sqrt{u^2+5u+4}}{24} + \frac{4u^2+5u+16}{24} - \frac{5(5^2-4(4))}{16}\ln\left|5+2u+2\sqrt{u^2+5u+4}\right|+C$$

$$-\frac{\sqrt{\cos^2\theta+5\cos\theta+4}}{24} + \frac{4\cos^2\theta+5\cos\theta+16}{24} + \frac{45}{16}\ln\left|5+2\cos\theta+2\sqrt{\cos^2\theta+5\cos\theta+4}\right|+C$$

Y. $$\int\frac{e^{3x}}{(e^{2x}-8)^{3/2}}\,dx$$

$u=e^x$
$du=e^x\,dx$

$$\int\frac{u^2\,du}{(u^2-8)^{3/2}}$$

$a=\sqrt{8}=2\sqrt{2}$

(3.48) $$\int\frac{x^2}{(x\pm a^2)^{3/2}}\,dx = \frac{-x}{\sqrt{x^2\pm a^2}} + \ln\left|x+\sqrt{x^2\pm a^2}\right|+C$$

$$\frac{-e^x}{\sqrt{e^{2x}-8}} + \ln\left|e^x+\sqrt{e^{2x}-8}\right|+C$$

zz. $\int \frac{1}{1 \pm \tan p}\, dp$

(4.37) $\int \frac{1}{1 \pm \tan x}\, dx = \frac{1}{2}\left[x \pm \ln|\cos x \pm \sin x|\right] + C$

$\Rightarrow \frac{1}{2}p \pm \frac{1}{2}\ln|\cos p + \sin p| + C$

aa. $\int e^{2x} \sinh\left(\frac{1}{2}e^{2x}\right) \sin\left(3e^{2x}\right) dx$ $\qquad u = e^{2x}$, $du = 2e^{2x}dx$, $\frac{1}{2}du = e^{2x}dx$

$\frac{1}{2}\int \sinh\left(\frac{1}{2}u\right)\sin(3u)\, du$

$a = \frac{1}{2}, b = 3$

(6.13) $\int \sinh(ax)\sin(bx)dx = \frac{1}{a^2+b^2}\left[a\cosh(ax)\sin(bx) - b\sinh(ax)\cos(bx)\right] + C$

$\Rightarrow \frac{1}{\frac{1}{4}+9}\left[\frac{1}{2}\cosh\left(\frac{1}{2}u\right)\sin(3u) - 3\sinh\left(\frac{1}{2}u\right)\cos(3u)\right] + C$

$= \frac{4}{1+36}\left[\frac{1}{2}\cosh\left(\frac{1}{2}e^{2x}\right)\sin\left(3e^{2x}\right) - 3\sinh\left(\frac{1}{2}e^{2x}\right)\cos\left(3e^{2x}\right)\right] + C$

$= \frac{2}{37}\cosh\left(\frac{1}{2}e^{2x}\right)\sin\left(3e^{2x}\right) - \frac{12}{37}\sinh\left(\frac{1}{2}e^{2x}\right)\cos\left(3e^{2x}\right) + C$

bb. $\int \ln^3(x-1)\, dx$ $\qquad u = x-1,\ du = dx$

(7.11) $\int \ln^n x\, dx = x\ln^n x - n\int \ln^{n-1} x\, dx + C$

$(x-1)\ln^3(x-1) - 3\int \ln^2 u\, du$

$(x-1)\ln^3(x-1) - 3\left[(x-1)\ln^2(x-1) - 2\int \ln u\, du\right.$

$(x-1)\ln^3(x-1) - 3(x-1)\ln^2(x-1) + 6(x-1)\ln(x-1) - 6(x-1) + C$

(7.1) $\int \ln(ax)dx = x\ln(ax) - x + C$

cc. $\int \frac{11}{2+3e^{t/10}}\, dt$ $\qquad u = t/10$, $10\, du = dt$ $\qquad 110\int \frac{1}{2+3e^{u}}\, du$

$k = 1, a = 2, b = 3$

(8.10) $\int \frac{1}{a+be^{kx}}\, dx = \frac{1}{ak}\left[kx - \ln|a+be^{kx}|\right] + C$

$\Rightarrow 110\left[\frac{1}{2}\left(u - \ln|2+3e^{u}|\right)\right] + C = \frac{110}{2}\cdot\frac{t}{10} - \frac{1}{2}\ln|2+3e^{t/10}| + C$

$\frac{11t}{2} - \frac{1}{2}\ln|2+3e^{t/10}| + C$

dd. $\int e^{-x}\cosh 2x\, dx \Rightarrow \int e^{ax}\cosh bx\, dx = \frac{e^{ax}}{a^2-b^2}\left[a\cosh bx - b\sinh bx\right] + C$ $\qquad a = -1, b = 2$, $a^2 \neq b^2$

$\frac{e^{-x}}{-3}\left[-1\cosh 2x - 2\sinh 2x\right] + C = \frac{e^{-x}}{3}\cosh 2x + \frac{2e^{-x}}{3}\sinh 2x + C$

8

2.

$$\int e^{2\theta}\sin 3\theta \, d\theta = \int e^{2\theta}\left(\frac{e^{3i\theta}-e^{-3i\theta}}{2i}\right)d\theta =$$

$$\frac{1}{2i}\int e^{(2+3i)\theta}-e^{(2-3i)\theta}\,d\theta = \frac{1}{2i}\left[\frac{1}{2+3i}e^{2+3i\theta}\left(\frac{2-3i}{2-3i}\right) - \frac{1}{2-3i}e^{(2-3i)\theta}\left(\frac{2+3i}{2+3i}\right)\right]+C$$

$$\frac{1}{2i}\left[\frac{2-3i}{4+9}e^{(2+3i)\theta} - \frac{2+3i}{4+9}e^{(2+3i)\theta}\right]+C$$

$$= \frac{1}{2i}\cdot\frac{e^{2\theta}}{13}\left[2e^{3i\theta}-3ie^{3i\theta}-2e^{-3i\theta}-3ie^{-3i\theta}\right]+C$$

$$\frac{e^{2\theta}}{13}\cdot\frac{1}{2i}\left[\left(2e^{3i\theta}-2e^{-3i\theta}\right)-3i\left(e^{3i\theta}+e^{-3i\theta}\right)\right]+C$$

$$\frac{e^{2\theta}}{13}\left[\frac{2\left(e^{3i\theta}-e^{-3i\theta}\right)}{2i} - \frac{3\cancel{i}}{2\cancel{i}}\left(e^{3i\theta}+e^{-3i\theta}\right)\right]+C$$

$$= \frac{e^{2\theta}}{13}\left[2\sinh(3\theta) - 3\cosh 3\theta\right]+C$$

3.a. $\int \operatorname{sech}^3 x \tanh x \, dx$ $\quad u = \operatorname{sech} x \quad du = -\operatorname{sech} x \tanh x \, dx$

$-\int u^2 \, du = -\frac{1}{3}u^3 + C \Rightarrow \boxed{-\frac{1}{3}\operatorname{sech}^3 x + C}$

b. $\int t \cot t \csc t \, dt$ $\quad u = t \quad du = dt \quad dv = \cot t \csc t \, dt \quad v = -\csc t$

$-t\csc t + \int \csc t \, dt = \boxed{-t\csc t - \ln|\csc t + \cot t| + C}$

c. $\int x \operatorname{arcsec} x \, dx$ $\quad u = \operatorname{arcsec} x \quad du = \frac{1}{x\sqrt{x^2-1}}dx \quad dv = x\,dx. \quad v = \frac{1}{2}x^2$

$\frac{1}{2}x^2 \operatorname{arcsec} x - \int \frac{1/2\, x^2}{x\sqrt{x^2-1}}dx = \frac{1}{2}x^2 \operatorname{arcsec} x - \frac{1}{2}\int \frac{x}{\sqrt{x^2-1}}dx$

$u = x^2-1 \quad du = 2x\,dx \quad \frac{1}{2}du = x\,dx \qquad \frac{1}{4}\int u^{-1/2}du \quad \frac{1}{4}(u^{1/2}$

$= \boxed{\frac{1}{2}x^2 \operatorname{arcsec} x - \frac{1}{2}\sqrt{x^2-1} + C.}$

3d. $\int_0^{\pi/3} \tan^2 x\, dx = \int_0^{\pi/3} \sec^2 x - 1\, dx = \tan x - x \Big|_0^{\pi/3} = \boxed{\sqrt{3} - \pi/3}$

e. $\int \cos^4 x\, dx = \int \left(\frac{1}{2}(1+\cos 2x)\right)^2 dx =$

$\frac{1}{4}\int 1 + 2\cos 2x + \cos^2 2x\, dx = \frac{1}{4}\int 1 + 2\cos 2x + \frac{1}{2}(1+\cos 4x)\, dx$

$= \frac{1}{4}\int \frac{3}{2} + 2\cos 2x + \frac{1}{2}\cos 4x\, dx = \boxed{\frac{1}{4}\left[\frac{3}{2}x + \sin 2x + \frac{1}{8}\sin 4x\right] + C}$

f. $\int \frac{\cot^3\theta\, d\theta}{\csc\theta} = \int \frac{\sin^3\theta\, d\theta}{\cos^3\theta\, d\theta} \cdot \sin\theta = \int \frac{\sin^4\theta}{\cos^3\theta}\, d\theta$

$= \int \frac{\sin^4\theta \cos\theta}{\cos^4\theta}\, d\theta = \int \frac{\sin^4\theta\cos\theta}{(1-\sin^2\theta)^2}\, d\theta \Rightarrow \begin{matrix} u = \sin\theta \\ du = \cos\theta\, d\theta \end{matrix}$

$\int \frac{u^4}{(1-u^2)^2}\, du = \frac{u}{2-2u^2} + u + \frac{3}{4}\ln(1-u) - \frac{3}{4}\ln(u+1) + C$

$= \frac{\sin\theta}{2 - 2\sin^2\theta} + \sin\theta + \frac{3}{4}\ln(1-\sin\theta) - \frac{3}{4}\ln(\sin\theta + 1) + C$

g. $\int e^{2x}\sqrt{1+e^{2x}}\, dx \qquad u = e^{2x} \quad du = 2e^{2x}dx$

$\frac{1}{2}\int \sqrt{1+u}\, du = \frac{2}{3}\cdot\frac{1}{2}(1+u)^{3/2} + C = \boxed{\frac{1}{3}(1+e^{2x})^{3/2} + C}$

h. $\int \frac{4x^2}{x^3+x^2-x-1}\, dx = \int \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$x^2(x+1) - (x+1) =$
$(x^2-1)(x+1)$
$(x-1)(x+1)(x+1)$

$A(x+1)^2 + B(x^2-1) + C(x-1)$
$Ax^2 + 2Ax + A + Bx^2 - B + Cx - C = 4x^2$

$A + B = 4 \qquad A = 1,\ B = 3,\ C = -2$
$2A + C = 0$
$A - B - C = 0$

$\int \frac{1}{x-1} + \frac{3}{x+1} - \frac{2}{(x+1)^2}\, dx = \boxed{\ln|x-1| + 3\ln|x+1| + \frac{2}{x+1} + C}$

i. $\int \frac{\sec^2 x}{\tan x(\tan x + 1)}\, dx \qquad \begin{matrix} u = \tan x \\ du = \sec^2 x \end{matrix} \qquad \int \frac{du}{u(u+1)} \qquad \begin{matrix} Au + A + Bu = 1 \\ A + B = 0 \\ A = 1 \to B = -1 \end{matrix}$

$\int \frac{A}{u} + \frac{B}{u+1}\, du = \int \frac{1}{u} - \frac{1}{u+1}\, du = \boxed{\ln|\tan x| - \ln|\tan x + 1| + C}$

3j. $\int x\sqrt{4-x}\,dx$ $u = x$, $du = dx$, $dv = (4-x)^{1/2}dx$, $v = -\frac{2}{3}(4-x)^{3/2}$

$-\frac{2}{3}x(4-x)^{3/2} + \frac{2}{3}\int(4-x)^{3/2}dx = -\frac{2}{3}x(4-x)^{3/2} - \frac{2}{3}\cdot\frac{2}{5}(4-x)^{5/2} + C$

$= \boxed{-\frac{2}{3}x(4-x)^{3/2} - \frac{4}{15}(4-x)^{5/2} + C}$

k. $\int 4\arccos x\,dx$ $u = \arccos x$, $du = \frac{-1}{\sqrt{1-x^2}}dx$, $dv = dx$, $v = x$

$4x\arccos x + 4\int\frac{x}{\sqrt{1-x^2}}dx$ $u = -x^2$, $du = -2x\,dx$, $-\frac{1}{2}du = x\,dx$ $-\frac{1}{2}\int(1+u)^{-1/2}du$ $-\frac{1}{2}(1+u)^{1/2}$

$\boxed{4x\arccos x - 4\sqrt{1-x^2} + C}$

l. $\int x^2(x-2)^{3/2}dx$

$\pm$	u	dv
$+$	x^2	$(x-2)^{3/2}$
$-$	$2x$	$\frac{2}{5}(x-2)^{5/2}$
$+$	2	$\frac{4}{35}(x-2)^{7/2}$
$-$	0	$\frac{8}{315}(x-2)^{9/2}$

$\boxed{\frac{2}{5}x^2(x-2)^{5/2} - \frac{8}{35}x(x-2)^{7/2} + \frac{16}{315}(x-2)^{9/2} + C}$

m. $\int \sin^5 x\cos^2 x\,dx \Rightarrow$ $u = \cos x$, $du = -\sin x\,dx$

$\sin x\,(1-\cos^2 x)^2\cos^2 x$

$-\int(1-u^2)^2u^2\,du = -\int(1-2u^2+u^4)u^2\,du = \int -u^6 + 2u^4 - u^2\,du$

$= \boxed{-\frac{1}{7}\cos^7 x + \frac{2}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C}$

n. $\int \sin x\tan^2 x\,dx = \int \sin x\cdot\frac{\sin^2 x}{\cos^2 x}dx = \int\frac{\sin x(1-\cos^2 x)}{\cos^2 x}dx$

$u = \cos x$, $du = -\sin x\,dx$

$\int\frac{u^2-1}{u^2}dx = \int 1 - u^{-2}\,du = u + \frac{1}{u} + C$

$= \boxed{\cos x + \sec x + C}$

30. $\int \frac{\sqrt{x^2+16}}{x}\,dx$ $\quad a=4$

(3.31) $\int \frac{\sqrt{a^2+x^2}}{x}\,dx = \sqrt{a^2+x^2} - a\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right| + C$

$\sqrt{16+x^2} - 4\ln\left|\frac{4+\sqrt{16+x^2}}{x}\right| + C$

p. $\int_3^6 \frac{\sqrt{x^2-9}}{x^2}\,dx$ $\quad a=3$

(3.33) $\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = -\frac{\sqrt{x^2-a^2}}{x} + \ln\left|x+\sqrt{x^2-a^2}\right| + C$

$= -\frac{\sqrt{x^2-9}}{x} + \ln\left|x+\sqrt{x^2-9}\right|\Big|_3^6 = -\frac{\sqrt{36-9}}{6} + \ln\left|6+\sqrt{36-9}\right| + 0 - \ln 3$

$= \frac{\cancel{3}\sqrt{3}}{\cancel{6}\,2} + \ln|6+3\sqrt{3}| - \ln 3$

$= \boxed{\frac{\sqrt{3}}{2} + \ln|6+3\sqrt{3}| - \ln 3}$

q. $\int_0^1 \frac{x^2-x}{x^2+x+1}\,dx$

$$\begin{array}{r|l} & 1 \\ x^2+x+1 & x^2-x \\ & -x^2\mp x\mp 1 \\ \hline & -2x-1 \end{array}$$

$\int_0^1 1 - \frac{2x+1}{x^2+x+1}\,dx$

$u = x^2+x+1$

$du = (2x+1)\,dx$

$x - \ln|x^2+x+1|\Big|_0^1$

$1 - \ln|3| - 0 = \boxed{1-\ln 3}$

r. $\int \frac{1}{t[1+(\ln t)^2]}\,dt$ $\quad u = \ln t$, $du = \frac{1}{t}\,dt$ $\quad \int \frac{1}{1+u^2}\,du$

$\arctan u + C = \boxed{\arctan(\ln t) + C}$

4a. $\lim_{x\to 0} \frac{\arcsin x}{x} = \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1} = 1$

c. $\lim_{x\to\infty} \frac{x^2}{\sqrt{x^2+1}} = \lim_{x\to\infty} \frac{\cancel{2x}}{\frac{1}{2}(x^2+1)^{-1/2}\cdot \cancel{2x}} = \lim_{x\to\infty} 2\sqrt{x^2+1} = \infty$

4b. $\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^{1/x} = (1+0)^0 = 1$

d. $\lim_{x\to 0^+} x^3 \cot x = \lim_{x\to 0^+} \frac{x^3}{\tan x} = \lim_{x\to 0^+} \frac{3x^2}{\sec^2 x} = 0$

5a. $\int_0^1 x^{-1/3}\,dx = \lim_{a\to 0}\int_a^1 x^{-1/3}\,dx = \lim_{a\to 0}\frac{3}{2}x^{2/3}\Big|_a^1 = \frac{3}{2}[1-0] = \frac{3}{2}$

converges

b. $\int_{-\infty}^1 e^x\,dx = \lim_{a\to -\infty}\int_a^1 e^x\,dx = \lim_{a\to -\infty} e^x\Big|_a^1 = \lim_{a\to -\infty} e^1 - e^a$

$= e$

converges

$u = \frac{1}{x}$ $1 \to 1$, $\infty \to q$

$du = -\frac{1}{x^2}\,dx$

c. $\int_0^1 \frac{e^{\frac{1}{x}}}{x^2}\,dx = \lim_{a\to 0}\int_a^1 \frac{e^{1/x}}{x^2}\,dx$

$= \lim_{q\to\infty}\int_q^1 -e^u\,du = \lim_{q\to\infty} -e^u\Big|_q^1 = \lim_{q\to\infty} -[e^1 - e^q] = \infty$

diverges

d. $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = \int_1^\infty 2e^{-u}\,du = \lim_{b\to\infty}\int_1^b 2e^{-u}\,du$

$u = \sqrt{x}$ $1 \to 1$, $\infty \to \infty$

$du = \frac{1}{2\sqrt{x}}\,dx$

$\to 2du = \frac{1}{\sqrt{x}}\,dx$

$= \lim_{b\to\infty} 2\left[\cancel{e^{-b}}^{\,0} - e^{-1}\right] = -\frac{2}{e}$ converges

e. $\int_2^\infty \frac{1}{\sqrt{x-1}}\,dx = \lim_{b\to\infty}\int_2^b (x-1)^{-1/2}\,dx = \lim_{b\to\infty} 2(x-1)^{1/2}\Big|_2^b =$

$\lim_{b\to\infty} 2(b-1)^{1/2} - 2(2-1)^{1/2} = \infty$ diverges

f. $\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx = \int_0^1 \frac{1}{\sqrt[3]{x-1}}\,dx + \int_1^9 \frac{1}{\sqrt[3]{x-1}}\,dx =$

$\lim_{b\to 1}\int_0^b (x-1)^{-1/3}\,dx + \lim_{a\to 1}\int_a^9 (x-1)^{-1/3}\,dx = \lim_{b\to 1}\frac{3}{2}(x-1)^{2/3}\Big|_0^b + \lim_{a\to 1}\frac{3}{2}(x-1)^{2/3}\Big|_a^9$

$\lim_{b\to 1}\left[\frac{3}{2}\cancel{(b-1)}^{\,0\,2/3} - \frac{3}{2}(0-1)^{2/3}\right] + \lim_{a\to 1}\left[\frac{3}{2}(9-1)^{2/3} - \frac{3}{2}\cancel{(a-1)}^{\,0\,2/3}\right] = -\frac{3}{2}(1) + \frac{3}{2}(8)^{2/3}$

$= 6 - \frac{3}{2} = \frac{9}{2}$

converges

5g. $\int_2^\infty \frac{1}{\sqrt[4]{1+x}}\,dx = \lim_{b\to\infty}\int_2^b (1+x)^{-1/4}\,dx = \lim_{b\to\infty}\frac{4}{3}(1+x)^{3/4}\Big|_2^b =$

$\lim_{b\to\infty} \frac{4}{3}(1+b)^{3/4} - \frac{4}{3}(1+2)^{3/4} = \infty$ diverges

h. $\int_1^\infty \frac{\ln x}{x}\,dx = \int_0^\infty u\,du = \lim_{b\to\infty}\int_0^b u\,du =$ $\quad u = \ln x$, $du = \frac{1}{x}\,dx$ $\quad \infty,\ 0$

$\lim_{b\to\infty} \frac{1}{2}u^2\Big|_0^b = \lim_{b\to\infty} \frac{1}{2}b^2 - 0 = \infty$ diverges

6. break into 3 cases, first $p=1$, $p\neq 1$ (2 cases)

$p=1$ $\int_1^\infty \frac{1}{x}\,dx = \ln x\Big|_1^\infty = \infty$ diverges

$p\neq 1$ $\int_1^\infty \frac{1}{x^p}\,dx = \int_1^\infty x^{-p}\,dx = \frac{x^{-p+1}}{-p+1}\Big|_1^\infty = \lim_{b\to\infty} \frac{b^{-p+1}}{-p+1} - \frac{1^{-p+1}}{-p+1}$

Split into 2 cases $\quad -p+1>0 \to -p>-1 \to p<1$

and $\quad -p+1<0 \to -p<-1 \to p>1$

for $p<1$ $\quad \lim_{b\to\infty} \frac{b^{-p+1}}{-p+1} - \frac{1^{-p+1}}{-p+1}$ $\quad$ diverges since ∞ to a positive power $\to \infty$

for $p>1$ $\quad \lim_{b\to\infty} \frac{b^{-p+1}}{-p+1} - \frac{1^{-p+1}}{-p+1}$ $\quad$ converges since ∞ to a negative power $\to 0$

So $p>1$ converges, $p\leq 1$ diverges

7. a. convert to all sines and cosines, then algebra.

b. trig substitution $x = \sin\theta$

c. by parts w/ borrowing x^2 each step

d. u-sub, power rule

e. change of variable / u-sub

7f. by parts may need to try $\int \frac{\sqrt{x}\arctan\sqrt{x}}{\sqrt{x}}\,dx$ w/ usub to progress

g. tables/ by parts may need to try $\int \frac{e^x\sqrt{1+e^x}}{e^x}\,dx$ w/ u sub

h. by parts or try arc-hyperbolic trig

i. u sub $u = \sec\theta$ then partial fractions

j. complete the square / arctangent

k. by parts to start, then may need partial fractions

l. change of variable

m. u-sub for $u = \ln x$, then trig sub $u = \tan\theta$

n. long division, u-sub

o. basic rule e^2 is a constant

p. product-to-sum formula / tables

q. multiply by $\frac{e^{-x}}{e^{-x}}$, u-sub

8a. problem at $x=1$ makes denom $=0$

$$\int_0^1 \frac{x}{x-1}\,dx + \int_1^9 \frac{x}{x-1}\,dx = \lim_{b\to1}\int_0^b \frac{x}{x-1}\,dx + \lim_{a\to1}\int_a^9 \frac{x}{x-1}\,dx$$

b. problem at $\pi/2$

$$\lim_{b\to\pi/2}\int_0^b \tan x\,dx + \lim_{a\to\pi/2}\int_a^\pi \tan x\,dx$$

c. problem at ∞ (-1 Not in interval)

$$\lim_{b\to\infty}\int_0^b \frac{1}{1+x^3}\,dx$$

$(x^2-x-2) = (x-2)(x+1)$

2 not in interval

d. problem at $x=-1$

$$\lim_{a\to-1}\int_a^1 \frac{dx}{x^2-x-2} = \lim_{a\to-1}\int_a^1 \frac{dx}{(x-2)(x+1)}$$ apply partial fractions