MTH 174, Exam #2, Spring 2018 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. All complex numbers should be stated in standard form, and all complex fractions should be simplified. If you do not show work, problems will be graded as "all or nothing" for the answer only; partial credit will not be possible and any credit awarded for the work will not be available.

1. Determine the type of conic by putting the equation $2x(x-y) = y(3-y-2x)$ in standard form. Sketch the graph. (10 points)

$2x^2 - \cancel{2xy} = 3y - y^2 - \cancel{2xy}$

$2x^2 + (y^2 - 3y + \frac{9}{4}) = \frac{9}{4}$

$\left[2(x^2) + (y-\frac{3}{2})^2 = \frac{9}{4}\right]\frac{4}{9}$

$\frac{8}{9}(x^2) + \frac{4}{9}(y-\frac{3}{2})^2 = 1$

$\frac{x^2}{(\frac{9}{8})} + \frac{(y-\frac{3}{2})^2}{(\frac{9}{4})} = 1$

ellipse

2. Sketch the parametric curves. Label the orientation. Convert the equations back to rectangular coordinates. (8 points each)

a. $x = t^3, y = 3\ln t$

$x = t^3, \ y = \ln t^3$

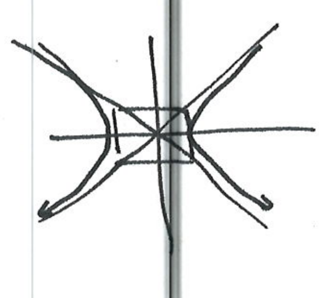
$y = \ln x$

b. $x = \sec t, y = \tan t$

$1 + \tan^2 t = \sec^2 t$

$1 + y^2 = x^2$

$1 = x^2 - y^2$



3. Find a parametric equation that passes through points $(1,4)$, *and* $(5,-2)$. (8 points)

$\langle 4, -6 \rangle$

$\vec{r}(t) = (4t+1)\hat{i} + (-6t+4)\hat{j}$

or

$x = 4t+1$
$y = 4-6t$

4. Find the equation of the tangent line for the parametric curve given by $x = 3t - t^2, y = 2t^{3/2}$ at $t = \frac{1}{4}$. (7 points)

$(11/16, 1/4)$

$\frac{dy}{dt} = 3t^{1/2} = 3\sqrt{t}$

$\frac{dx}{dt} = 3-2t$

$\frac{dy}{dx} = \frac{3\sqrt{t}}{3-2t} \quad t = 1/4 \rightarrow \frac{3}{5}$

$y - \frac{1}{4} = \frac{3}{5}(x - 11/16)$

5. Find the equation(s) of the tangent line(s) to the graph $r = 3\cos 2\theta$, passing through the pole. (10 points)

$0 = 3\cos 2\theta$
$\cos 2\theta = 0$
$2\theta = \pi/2, 3\pi/2$
$\theta = \pi/4, 3\pi/4$

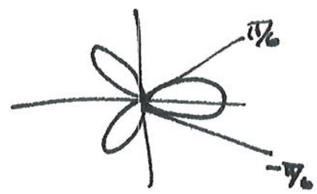
$y = x$ and $y = -x$

$\frac{dr}{d\theta} = -6\sin 2\theta$

$\frac{dy}{dx} = \frac{-6\sin 2\theta \cdot \sin\theta + 3\cos 2\theta \cdot \cos\theta}{-6\sin 2\theta \cdot \cos\theta - 3\cos 2\theta \cdot \sin\theta} = 1$ at $\pi/4$, -1 at $3\pi/4$

6. Find the area of one petal of $r = 4\cos 3\theta$. (12 points)

$$\frac{1}{2}\int_{-\pi/6}^{\pi/6} (4\cos 3\theta)^2 \, d\theta =$$



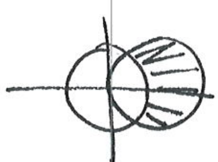
$$= 4\int_{-\pi/6}^{\pi/6} 1 + \cos 6\theta \, d\theta =$$

$$4\left[\theta + \frac{1}{6}\cancel{\sin 6\theta}^{\,0}\right|_{-\pi/6}^{\pi/6} = \frac{4\pi}{3}$$

$4\cos 3\theta = 0$ $\quad 3\theta = \pi/2, -\pi/2$

$\cos 3\theta = 0$ $\quad \theta = \pi/6, -\pi/6$

7. Find the area inside $r = 2\cos\theta$, and outside $r = 1$. (15 points)



$1 = 2\cos\theta$

$\frac{1}{2} = \cos\theta$

$\theta = \pi/3, -\pi/3$

$$\frac{1}{2}\int_{-\pi/3}^{\pi/3} (2\cos\theta)^2 - 1^2 \, d\theta =$$

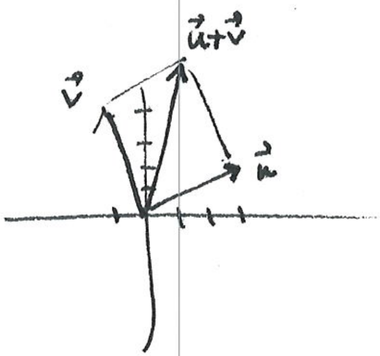
$$\frac{1}{2}\int_{-\pi/3}^{\pi/3} 4\cos^2\theta - 1 \, d\theta = \frac{1}{2}\int_{-\pi/3}^{\pi/3} 2 + 2\cos 2\theta - 1 \, d\theta$$

$$\frac{1}{2}\int_{-\pi/2}^{\pi/3} 1 + 2\cos 2\theta \, d\theta = \frac{1}{2}\left[\theta + \sin 2\theta\right|_{-\pi/3}^{\pi/3} =$$

$$\frac{1}{2}\left[\pi/3 + \pi/3 + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right] = \frac{\pi}{3} + \frac{\sqrt{3}}{2}$$

8. Sketch the vectors $\vec{u} = \langle 3,2 \rangle, \vec{v} = \langle -1,4 \rangle$ along with $\vec{u} + \vec{v}$. Use the graph to illustrate the parallelogram rule. (7 points)

$\vec{u}+\vec{v} = \langle 2, 6 \rangle$



9. Use $\vec{u} = \langle 4,3 \rangle, \vec{v} = \langle 12,-5 \rangle$ to find the following. (5 points each)
   a. $\vec{u} + \vec{v}$

$\langle 16, -2 \rangle$

   b. $||\vec{u}||$

$\sqrt{16+9} = \sqrt{25} + 5$

   c. Write $\vec{v}$ in polar form.

$||\vec{v}|| = \sqrt{144+25} = \sqrt{169} = 13$

$\tan^{-1}\left(-\frac{5}{12}\right) = -.395$ radians or $-22.6°$

$\vec{v} = 13\cos(-.395)\hat{i} + 13\sin(-.395)\hat{j}$

d. Write a unit vector in the direction of $\vec{u}$

$\hat{u} = \langle \frac{4}{5}, \frac{3}{5} \rangle$

e. Find $\vec{u} \cdot \vec{v}$

$48 - 15 = 33$

f. Find the angle between $\vec{u}$ and $\vec{v}$

$\cos^{-1}\left(\frac{u \cdot v}{\|u\|\|v\|}\right) = \cos^{-1}\left(\frac{33}{5 \cdot 13}\right) \approx 1.038$ radians

or $59.49°$

10. Find the resulting for$\overset{ce}{\swarrow}$ and direction of adding $\|F_1\| = 75\ lbs., \theta_1 = 30°, \|F_2\| = 100\ lbs., \theta_2 = 45°, and\ \|F_3\| = 125\ lbs., \theta_3 = 120°$. Round answers to one decimal place. (10 points)

$F_1 = \langle 75\cos 30°, 75\sin 30° \rangle = \langle 64.952, 37.5 \rangle$

$F_2 = \langle 100\cos 45°, 100\sin 45° \rangle = \langle 50\sqrt{2}, 50\sqrt{2} \rangle$

$F_3 = \langle 125\cos 120°, 125\sin 120° \rangle = \langle -62.5, 108.25 \rangle$

$F_{TOTAL} = \langle 73.16, 216.46 \rangle$

$\|F_T\| = 228.49$

$\tan^{-1}\left(\frac{216.46}{73.16}\right) = 71.33°$

magnitude 228.5 lbs

direction 71.3°

11. Find the work done by pulling a car with a force of 1600 newtons at an able of 25° (with respect to the horizontal) if the wagon is pulled 2 kilometers. Round answer to one decimal place. (7 points)

$F = \langle 1600 \cos 25°, 1600 \sin 25° \rangle$

$d = \langle 2, 0 \rangle$

$F \cdot d = 2900.2$ newton · km

Useful formulas:

$$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$