MAT 230, Paper Quiz #7, Spring 2017 Name KEY

Instructions: Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations.

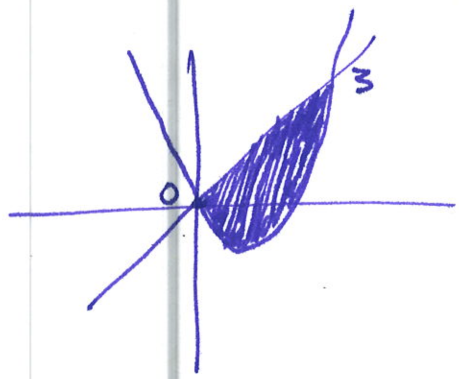
1. Find the area between the curves $y = x^2 - 2x, y = x$. Sketch the graphs.

$\int_0^3 x - (x^2 - 2x)\, dx =$

$\int_0^3 3x - x^2\, dx =$

$\frac{3}{2}x^2 - \frac{1}{3}x^3 \Big|_0^3 =$

$\frac{3}{2}(3)^2 - \frac{1}{3}(3)^3 = \frac{27}{2} - 9 = \frac{9}{2}$



2. Integrate.

a. $\int \frac{dx}{1-x}$

$u = 1-x$
$du = -dx$
$-du = dx$

$\int -\frac{1}{u}\, du = \int -u^{-1}\, du = -\ln u + C$
$= -\ln(1-x) + C$
$= \ln\left(\frac{1}{1-x}\right) + C$

b. $\int \frac{(\ln x)^3}{x} dx$

$u = \ln x$
$du = \frac{1}{x} dx$

$\int u^3\, du = \frac{1}{4}u^4 + C$
$= \frac{1}{4}(\ln x)^4 + C$

c. $\int x\sqrt{x-3}\, dx$

$u = \sqrt{x-3}$
$u^2 = x - 3$
$u^2 + 3 = x$
$2u\,du = dx$

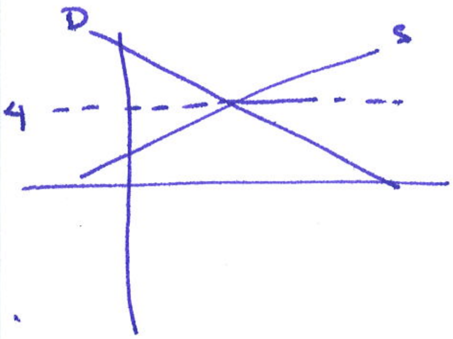
$\int (u^2 - 3)u \cdot 2u\, du =$
$\int 2u^2(u^2 - 3)\, du =$
$\int 2u^4 - 6u^2\, du$
$= \frac{2}{5}u^5 - 2u^3 + C$
$= \frac{2}{5}(x-3)^{5/2} - 2(x-3)^{3/2} + C$

3. Find the producer's and consumer's surplus for the demand function $D(x) = -\frac{5}{6}x + 9$, and the supply function $S(x) = \frac{1}{2}x + 1$.

$$-\frac{5}{6}x+9=\frac{1}{2}x+1$$
$$\quad -1 \qquad\qquad -1$$
$$+\frac{5}{6}x \qquad +\frac{5}{6}x$$

$$8=\frac{4}{3}x$$

$$x=6 \qquad\qquad -\frac{5}{6}(6)+9=4$$



$$\int_0^6 -\frac{5}{6}x+9-4\,dx = \int_0^6 -\frac{5}{6}x+5\,dx =$$

$$-\frac{5}{12}x^2+5x\Big|_0^6 = -\frac{5}{12}(36)+30=15$$ consumer's surplus

$$\int_0^6 4-(\tfrac{1}{2}x+1)\,dx = \int_0^6 4-\tfrac{1}{2}x-1\,dx = \int_0^6 3-\tfrac{1}{2}x\,dx = 3x-\frac{1}{4}x^2\Big|_0^6 =$$

$$18-9=9$$ producer's surplus

4. Find y if $y' = x^{2/3} - x, y(1) = -6$

$$y=\int x^{2/3}-x\,dx = \frac{3}{5}x^{5/3}-\frac{1}{2}x^2+C$$

$$\frac{3}{5}(1)^{5/3}-\frac{1}{2}(1)^2+C=-6$$

$$\frac{1}{10}+C=-6$$

$$C=\frac{[illegible]}{10}=6.1$$

$$y=\frac{3}{5}x^{5/3}-\frac{1}{2}x^2-6.1$$