MAT 230 Written Homework #4 Key

1. a. $y' = \frac{-12x^2}{5} e^{x^3}$

b. $f(x) = \ln(x^2-7) - \ln x \quad f'(x) = \frac{2x}{x^2-7} - \frac{1}{x}$

c. $h(x) = \left(x^2 + (1-3x)^{1/2}\right)^{1/2}$

$h'(x) = \frac{1}{2}\left(x^2 + (1-3x)^{1/2}\right)^{-1/2}\left(2x + \frac{1}{2}(1-3x)^{-1/2}(-3)\right)$

d. $g(x) = e^{3x}x^{-6} \quad g'(x) = 3e^{3x}x^{-6} - 6e^{3x}x^{-7}$

e. $p(x) = (\ln x)^3 \ln[\ln(e^{x^2}+6)]$

$p'(x) = 3(\ln x)^2 \cdot \frac{1}{x} \ln[\ln(e^{x^2}+6)] + (\ln x)^3 \frac{1}{\ln(e^{x^2}+6)} \cdot \frac{1}{e^{x^2}+6} \cdot e^{x^2} \cdot 2x$

2. y''', f''', $\frac{d^3y}{dx^3}$, $\frac{d^3f}{dx^3}$, $\dddot{y}$, $f^{(3)}$, $D_x^3[f(x)]$

answers may vary

3. $f(x) = x^6 - x^3 - \frac{2}{x} = x^6 - x^3 - 2x^{-1}$

$f'(x) = 6x^5 - 3x^2 + 2x^{-2}$

$f''(x) = 30x^4 - 6x - 4x^{-3} = 30x^4 - 6x - \frac{4}{x^3}$

4. $f(x) = \sqrt[4]{x} - \sqrt{x} = x^{1/4} - x^{1/2}$

$f'(x) = \frac{1}{4}x^{-3/4} - \frac{1}{2}x^{-1/2}$

$f''(x) = \frac{-3}{16}x^{-7/4} + \frac{1}{4}x^{-3/2}$

$f'''(x) = \frac{21}{64}x^{-11/4} - \frac{3}{8}x^{-5/2}$

$f^{IV}(x) = \frac{-231}{256}x^{-15/4} + \frac{15}{16}x^{-7/2}$

$f^{V}(x) = \frac{3465}{1024}x^{-19/4} - \frac{105}{32}x^{-9/2}$

5a. $\lim_{x \to -2} \left(\frac{x^2-4}{x+2} \right) = \lim_{x \to -2} \frac{2x}{1} = -4$

$\frac{0}{0}$ case

b. $\lim_{x \to \infty} \frac{3x^3+x+11}{6x^3+x+2} = \lim_{x \to \infty} \frac{9x^2+1}{18x^2+1} = \lim_{x \to \infty} \frac{18x}{36x} = \frac{1}{2}$

$\frac{\infty}{\infty}$ case $\frac{\infty}{\infty}$ case