MAT 230 Homework #2 Key

1a. $\frac{(x+h)^2-(x+h)-(x^2-x)}{h} = \frac{x^2+2xh+h^2-x-h-x^2+x}{h} =$

$\frac{2xh+h^2-h}{h} = \frac{h(2x+h-1)}{h} = 2x+h-1$

b. $\frac{(x+h)^3-x^3}{h} = \frac{x^3+3x^2h+3xh^2+h^3-x^3}{h} = \frac{3x^2h+3xh^2+h^3}{h}$

$= \frac{h(3x^2+3xh+h^2)}{h} = 3x^2+3xh+h^2$

c. $\frac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h} = \frac{1}{h}\left[\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x+h}\cdot\sqrt{x}}\right] = \frac{1}{h}\left[\frac{(\sqrt{x}-\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}{(\sqrt{x+h}\cdot\sqrt{x})(\sqrt{x}+\sqrt{x+h})}\right] =$

$\frac{1}{h}\left[\frac{x-x-h}{(\sqrt{x}\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}\right] = \left[\frac{-1}{(\sqrt{x}\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}\right]$

d. $\frac{\frac{1}{x+h+9}-\frac{1}{x+9}}{h} = \frac{1}{h}\left[\frac{x+9-x-h-9}{(x+9)(x+h+9)}\right] = \frac{1}{h}\left[\frac{-h}{(x+9)(x+h+9)}\right] =$

$\frac{-1}{(x+9)(x+h+9)}$

2. a. $\lim_{h\to 0} \frac{-2(x+h)^2+2x^2}{h} = \lim_{h\to 0} \frac{-2x^2-4xh-2h^2+2x^2}{h} = \lim_{h\to 0} \frac{-4xh-2h^2}{h}$

$= \lim_{h\to 0} \frac{h(-4x-2h)}{h} = \lim_{h\to 0} -4x-2h = -4x = f'(x)$

b. $f'(1) = -4$

c. $f(1) = -2(1)^2 = -2 \quad (1,-2)$

$y+2 = -4(x-1) \Rightarrow y+2 = -4x+4 \Rightarrow y = -4x+2$

$\underline{-2} \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underline{-2}$

3. a. answers may vary

b.