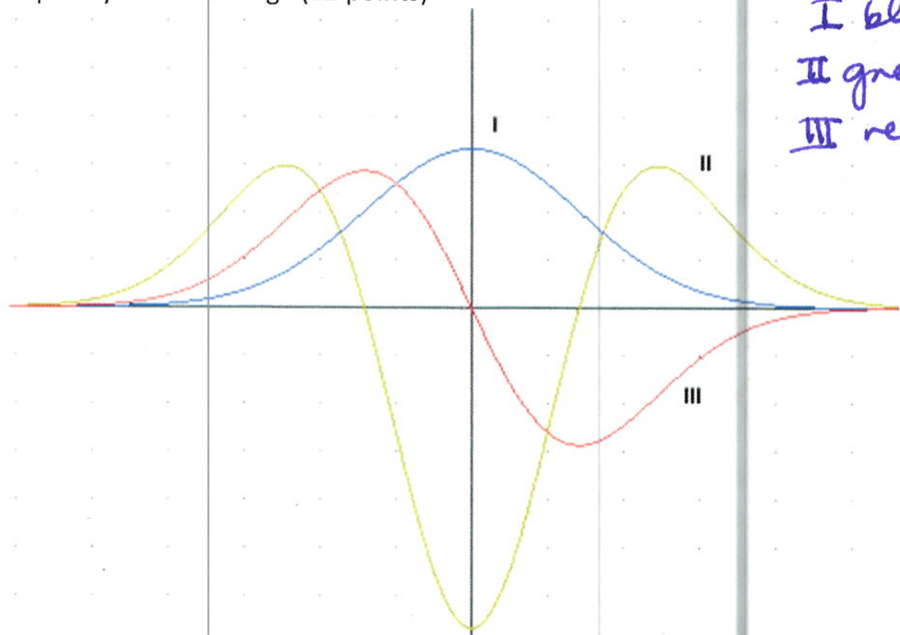
MAT 230, ~~Exam #2~~ Final Exam, Part I, Spring 2017 Name KEY

Instructions: Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations. On this portion of the exam, **no calculator is permitted**.

1. The graph below shows three functions. One function $f'(x)$ is the derivative of the other function $f(x)$, and the third function is $f''(x)$. Determine which graph is which, and label each. Explain your reasoning. (12 points)



I blue line is $f(x)$
II green line is $f''(x)$
III red line is $f'(x)$

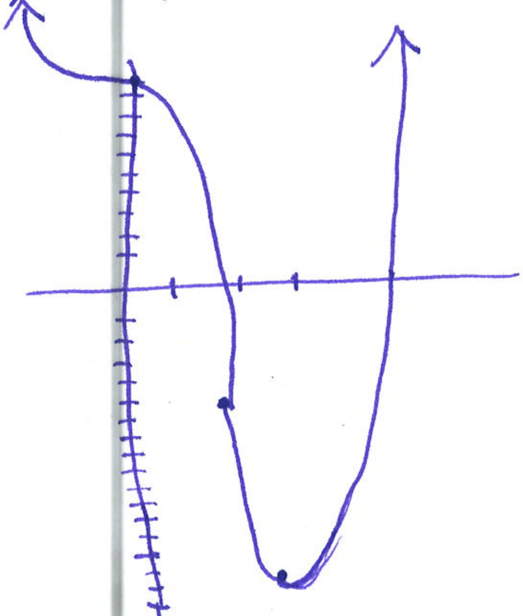
2. Consider the function $f(x) = x^4 - 4x^3 + 10$. Final all the extrema and inflection points. Classify all extrema. Use that information to sketch the curve. (15 points)

$f'(x) = 4x^3 - 12x^2 = 4x^2(x-3) \quad x=0, x=3$

$f''(x) = 12x^2 - 24x = 12x(x-2) \quad x=0, x=2$

f': $-$ | 0 | $-$ | 3 | $+$

f'': $-$ | 0 | $+$ | 2 | $-$



$f(0) = 10$

$f(2) = -6$

$f(3) = -17$

3. Find $\frac{dy}{dx}$ for $y^5\sqrt{x} = 96$. Use it to find the slope of the tangent line at (9,2). (15 points)

$$y^5 x^{1/2} = 96$$

$$5y^4 y' x^{1/2} + y^5 \cdot \frac{1}{2} x^{-1/2} = 0$$

$$5y^4 x^{1/2} y' = -\frac{1}{2} y^5 x^{-1/2}$$

$$y' = \frac{-\frac{1}{2} y^5 x^{-1/2}}{5y^4 x^{1/2}} = \frac{-y}{10x}$$

$$\text{Slope} = \frac{-2}{90} = \frac{-1}{45}$$

$$y-2 = \frac{-1}{45}(x-9) \Rightarrow$$

$$y-2 = -\frac{1}{45}x + \frac{1}{5} \Rightarrow y = \frac{-1}{45}x + \frac{11}{5}$$

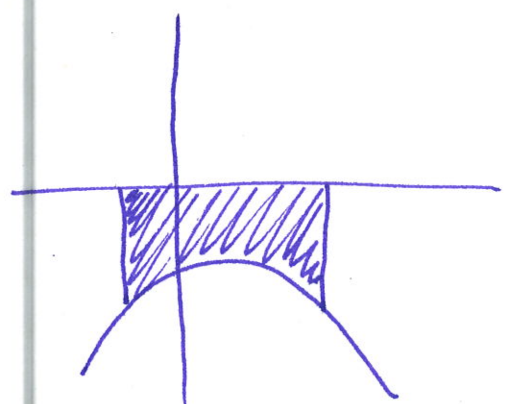
4. Evaluate $\int_{-2}^{3} -3x^2 + 4x - 7\, dx$. (12 points)

$$-\frac{3}{3}x^3 + 2x^2 - 7x \Big|_{-2}^{3}$$

$$-(3)^3 + 2(9) - 7(3) - [-(-2)^3 + 2(-2)^2 - 7(-2)] =$$

$$-27 + 18 - 21 - [8 + 8 + 14] =$$

$$-30 - [30] = -60$$



5. Evaluate $f(x, y, z) = 2^x + 5zy - x$ at $(0,1,-3)$ and $(1,0,-3)$. (8 points)

$$f(0,1,-3) = 2^0 + 5(-3)(1) - 0 = 1 - 15 - 0 = -14$$

$$f(1,0,-3) = 2^1 + 5(-3)(0) - 1 = 2 - 0 - 1 = 1$$

6. Find the domain of the functions. (7 points each)

a. $g(x, y) = \ln(x - y)$

$X - Y > 0$

$X > Y$

$D: \{(x,y) \mid X > Y\}$

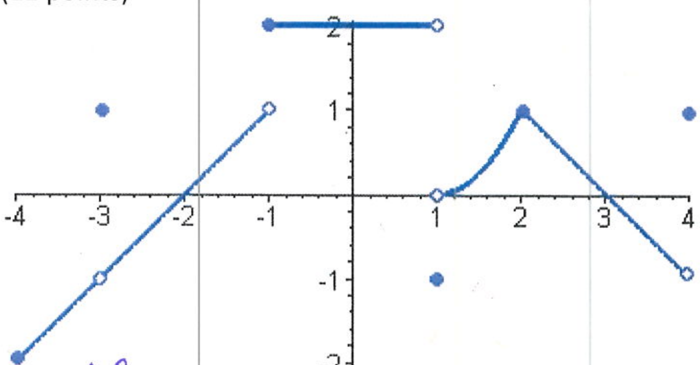
b. $g(x, y) = \frac{1}{y+x^2}$

$Y + X^2 \neq 0$

$X^2 \neq -Y$

$D: \{(x,y) \mid Y \neq -X^2\}$

7. Consider the graph of the piecewise function $f(x)$ below. Describe all points of discontinuity (explain your reasoning). Note any places where the graph is continuous but not differentiable. (12 points)



Continuity problems:

hole at (-3) limit exists but $\neq f(-3)$

jump at $x = -1$, limit does not exist

jump at $x = 1$, limit does not exist

hole at $x = 4$ left hand limit not equal to $f(4)$

Continuous at all other points in the domain $[-4, 4]$

8. Find the limit $\lim_{x \to 1} \frac{x^3-1}{x-1}$. Use any method (algebra or L'Hôpital's). (10 points)

$\frac{0}{0}$

$$= \lim_{x \to 1} \frac{3x^2}{1} = 3$$

9. Use the limit definition of the derivative to find $f'(x)$ for $f(x) = 3x - x^2$. (15 points)

$$\lim_{h \to 0} \frac{3(x+h) - (x+h)^2 - (3x - x^2)}{h} =$$

$$\lim_{h \to 0} \frac{\cancel{3x} + 3h - \cancel{x^2} - 2xh - h^2 - \cancel{3x} + \cancel{x^2}}{h} =$$

$$\lim_{h \to 0} \frac{3h - 2xh - h^2}{h} = \lim_{h \to 0} \frac{\cancel{h}(3 - 2x - h)}{\cancel{h}}$$

$$= \lim_{h \to 0} 3 - 2x - h = 3 - 2x$$

Final Exam
MAT 230, ~~Exam #2~~, Part II, Spring 2017 Name KEY

Instructions: Show all work, and provide exact answers. For full credit will be given to the steps shown than for the final answer. Be sure to provide thorough explanations. You may use a calculator on this portion of the exam. If you use your calculator, describe the steps you used, or sketch the graph obtained from your calculator to show work.

1. Find f_x and f_y for $f(x, y) = e^{xy}$. (10 points)

$f_x = ye^{xy}$

$f_y = xe^{xy}$

2. Find all extrema for $f(x, y) = 4xy - x^3 - 2y^2$ and classify all the points as maxima, minima, or saddle points (or say why it's not possible). (20 points)

$f_x = 4y - 3x^2 = 0 \Rightarrow 4y = 3x^2$

$f_y = 4x - 4y = 0 \Rightarrow 4x = 4y \Rightarrow x = y$

$4x = 3x^2 \Rightarrow 3x^2 - 4x = 0$

$x(3x-4) = 0$

$x = 0,\ x = 4/3$

$y = 0\quad y = 4/3$

$f_{xx} = -6x$

$f_{yy} = -4$

$f_{xy} = 4$

$D(0,0) = (0)(-4) - 4^2 = -16$ saddle point

$D(4/3, 4/3) = -6(\frac{4}{3})(-4) - 16 = 16$ $f_{xx}, f_{yy} < 0$ max.

3. McLeod Corps finds that its profit P, in millions of dollars, is given by $P(a,p) = 2ap + 80p - 15p^2 - \frac{1}{10}a^2 - 80$, where a is amount spent on advertising, and p is price per unit in dollars. Find the maximum value of P and the values of a and p at where it occurs. (15 points)

$P_a = 2p - \frac{1}{5}a = 0 \Rightarrow 2p = \frac{1}{5}a \Rightarrow 10p = a$

$P_p = 2a + 80 - 30p = 0 \Rightarrow 3(10p) = 2a + 80 \Rightarrow 3a = 2a + 80$

$a = 80$

$p = 8$

$P_{aa} = -\frac{1}{5}$ $\quad D = (-\frac{1}{5})(-30) - 2^2 =$

$P_{ap} = 2$ $\quad 6 - 4 = 2$ max or min

$P_{pp} = -30$ both neg $\cap$ $\boxed{\text{max}}$

$P(80, 8) = 240$ million

4. Find a regression equation to model the data below. (15 points)

Years since 1990, x	0	10	13	17	21
Life expectancy of men, y	71.8	74.1	74.8	75.4	76.3

a. Write the equation.

$y = .2128x + 71.88$

b. Use the equation to predict the life expectancy of men (in years) in 2020 and 2025.

$y(30) = 78.3$

$y(35) = 79.3$

5. Find the absolute extrema of the function $f(x) = x + \frac{1}{x}$ on the interval $[1,20]$. (10 points)

$f' = 1 - \frac{1}{x^2} = 0 \quad x = \pm 1$ $\quad -1$ outside interval

Check 1, 20

$f(1) = 2 \leftarrow$ abs min

$f(20)\ 20.05 \leftarrow$ abs max

6. Integrate.

a. $\int 4\sqrt[3]{x^5} + \frac{3}{7}e^{-x} - \frac{7}{x^2}\, dx$ (10 points)

$\int 4x^{5/3} + \frac{3}{7}e^{-x} - 7x^{-2}\, dx$

$= 4\left(\frac{3}{8}\right)x^{8/3} - \frac{3}{7}e^{-x} + 7x^{-1} + C$

$\frac{3}{2}\sqrt[3]{x^8} - \frac{3}{7}e^{-x} + \frac{7}{x} + C$

b. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} - \frac{(\ln x)^2}{x} + (6x^2 - 1)(2x^3 - x + 9)^3\, dx$ (15 points)

$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$ $\quad u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$2du = \frac{1}{\sqrt{x}}$

$\int 2e^u\, du = 2e^u$

$\int \frac{(\ln x)^2}{x}\, dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$ $\quad \int u^2\, du = \frac{1}{3}u^3$

$\int (6x-1)(2x^3 - x + 9)^3\, dx$

$u = 2x^3 - x + 9$

$du = 6x^2 - 1\, dx$

$\int u^3\, du = \frac{1}{4}u^4 + c$

$2e^{\sqrt{x}} - \frac{(\ln x)^3}{3} + \frac{1}{4}(2x^3 - x + 9)^4 + C$

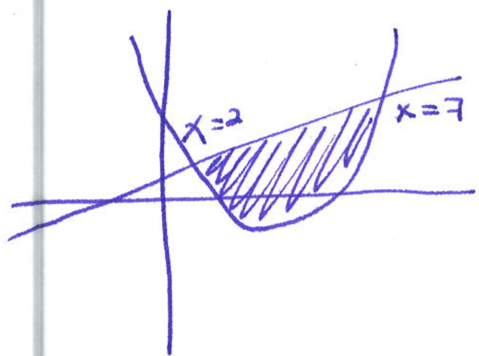
7. Set up an integral to find the area between the curves $f(x) = x^2 - 7x + 20, g(x) = 2x + 6$. You do not need to evaluate it. Sketch the graph. (10 points)

$x^2 - 7x + 20 = 2x + 6$

$x^2 - 9x + 14 = 0$

$(x-2)(x-7) = 0$

$x = 2, 7$



$$\int_2^7 (2x+6) - (x^2 - 7x + 20)\, dx$$

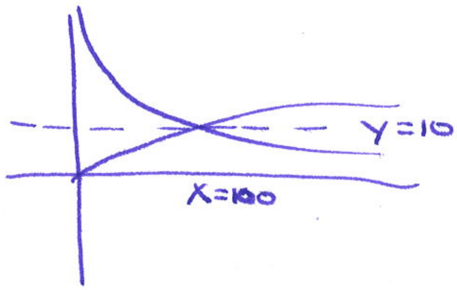
$$= \int_2^7 2x + 6 - x^2 + 7x - 20\, dx$$

$$= \int_2^7 9x - 14 - x^2\, dx$$

8. If the demand function is $D(x) = \frac{100}{\sqrt{x}}$, and the supply function is $S(x) = \sqrt{x}$, set up the integrals for consumer's and producer's surplus. You do not need to evaluate them. (15 points)

$\frac{100}{\sqrt{x}} = \sqrt{x} \Rightarrow 100 = x$

$\frac{100}{\sqrt{100}} = \frac{100}{10} = 10$



Consumer's surplus

$$\int_0^{100} \frac{100}{\sqrt{x}} - 10\, dx$$

producer's surplus

$$\int_0^{100} 10 - \sqrt{x}\, dx$$

9. Find y if $\frac{dy}{dx} = \frac{3x}{y^2}$, and $y(0) = -4$. (12 points)

$$y^2 dy = 3x\,dx$$
$$\frac{1}{3}y^3 = \frac{3}{2}x^2 + C$$
$$y^3 = \frac{9}{2}x^2 + C$$
$$y = \sqrt[3]{\frac{9}{2}x^2 + C}$$
$$-4 = y = \sqrt[3]{0 + C}$$

$$(-4)^3 = -64 = C$$
$$y = \sqrt[3]{\frac{9}{2}x^2 - 64}$$

10. Use the product, quotient or chain rule to differentiate. You do not need to simplify. (10 points each)

a. $f(t) = (3t^5 - t^2)\left(t - \frac{5}{t}\right)$

$$(15t^4 - 2t)\left(t - \frac{5}{t}\right) + (3t^5 - t^2)\left(1 + \frac{5}{t^2}\right)$$

b. $f(x) = \frac{x-1}{x+x^{-2}}$

$$\frac{1(x + x^{-2}) - (1 - 2x^{-3})(x-1)}{(x + x^{-2})^2}$$

c. $f(x) = (3\ln x + x^2)^4 + \frac{11}{13}e^{-x^e + 4x}$

$$4(3\ln x + x^2)^3\left(\frac{3}{x} + 2x\right) + \frac{11}{13}e^{-x^e + 4x}(-ex^{e-1} + 4)$$