

While we normally calculate expected values from a probability distribution and a random variable, to understand what is going on, we can put problems in context and create an appropriate set of random variables and probabilities from the other end.

You are part of a PTA committee that wants to get 50 new tablets for the school for use in Mathematics and Science courses. The school doesn't have the budget for it, so the PTA decides to hold a charity raffle to raise funds for the purchase.

First, we need to determine how much money is needed.

1. If each tablet costs approximately \$100, find how much money the school would have to raise from the raffle to afford all 50 tablets.

$$50 \times 100 = \$5,000$$

2. Suppose that the PTA decides to give away the following prizes to induce members of the community to purchase raffle tickets:
  - a. First place prize: \$1000
  - b. Second place prize: \$500
  - c. Third place prize: \$50 (you will offer 5 of these)
  - d. Fourth place prize: A gift certificate worth \$10 to school book store (you will offer 10 of these)

Use this information to calculate the total amount to be allocated for prize money.

$$1000 + 500 + 5 \times 50 + 10(10) = \$1,850$$

3. You decide that you are going to rent out a room to hold a party to give away the raffle prizes. After some research, you find that you can use the gymnasium, and volunteers to set up the event, and catering for food will run around \$2500. This cost will also need to be covered by the cost of raffle tickets. Adding this cost to the previous two values, what is the total amount of money needed to cover the party, the tablets and the prizes.

$$5000 + 1850 + 2500 = 9350$$

Then, we need to determine, how many tickets need to be sold, and at what cost, to cover all the expenses listed, and still have enough to achieve the goal.

4. Suppose that the PTA leadership believes it can sell 500 raffle tickets for the event. How much would have to be charged per raffle ticket to cover the costs listed above?

$$\frac{9350}{500} = \$18.70$$

5. What if the PTA is able to sell 750 tickets? How much would they have to sell each ticket for then?

$$\frac{9350}{750} = \$12.47$$

6. How many tickets would they have to sell if they only wanted to charge \$10 per ticket?

$$\frac{9350}{10} = 935$$

7. For each of the scenarios in 4-6, create an expected value table for the purchasers of each ticket and calculate the expected value of the ticket. What do you notice about the expected value in each case? How does it relate to the previous calculations?

|              |                     |                    |                  |                  |         |  |
|--------------|---------------------|--------------------|------------------|------------------|---------|--|
| X            | 981.3<br>1000-18.70 | 481.3<br>500-18.70 | 31.3<br>50-18.70 | -8.7<br>10-18.70 | -18.70  | when multiplied<br>by # of tickets<br>all E(X)'s<br>will cover<br>non prize<br>costs for<br>tablets &<br>catering. |
| P(x)         | 1/500               | 1/500              | 5/500            | 10/500           | 483/500 |  |
| $E(X) = -15$ |                     |                    |                  |                  |         | $15 \times 500 = 7500$   |

|      |                      |                     |                   |                   |         |
|------|----------------------|---------------------|-------------------|-------------------|---------|
| X    | 987.53<br>1000-12.47 | 487.53<br>500-12.47 | 37.53<br>50-12.47 | -2.47<br>10-12.47 | -12.47  |
| P(x) | 1/750                | 1/750               | 5/750             | 10/750            | 733/500 |

$$E(X) = -10.00 \quad 10 \times 750 = 7500$$

|      |                |               |             |            |         |
|------|----------------|---------------|-------------|------------|---------|
| X    | 990<br>1000-10 | 490<br>500-10 | 40<br>50-10 | 0<br>10-10 | -10     |
| P(x) | 1/935          | 1/935         | 5/935       | 10/935     | 918/935 |

$$802 \times 935 = \$7498.70$$

$$E(X) = -8.02$$