MAT 223, Exam #2, Spring 2017 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Explain thoroughly using complete sentences. If you use your calculator to perform statistical tasks, say which command/operations you are using and what you entered into your calculator, and what you got back to show work. If you do not show work and the answer is incorrect, no credit will be awarded.

1. For each of the probabilities below, label each as subjective, experimental or classical. (2 points each)
   a. A 20-sided die has faces numbered 1 through 20. The probability that a number divisible by 5 will come up is 1/5.

   theoretical (classical)

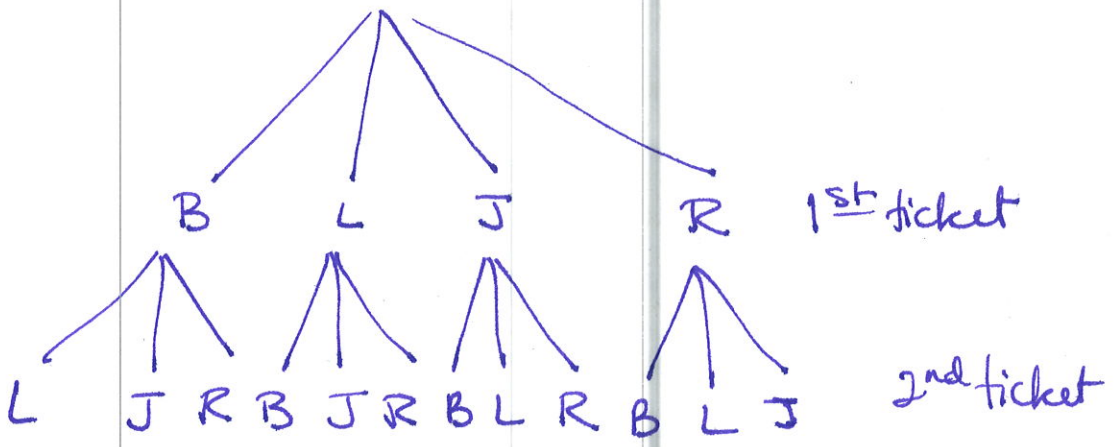
   b. The probability of a nuclear war increased in the last six months.

   Subjective (personal)

   c. A coin is flipped 100 times and heads came up 42% of the time, so the probability of getting a heads is 0.42.

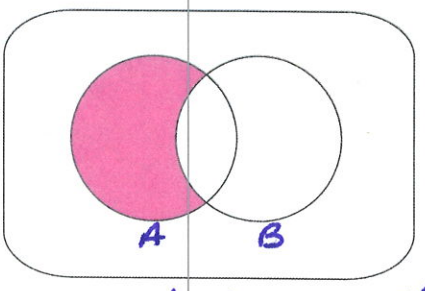
   experimental (empirical)

2. Pam has four friends (Barbara, Lela, Jo, and Roxie) but has only two tickets to the Rock-n-Roll Hall of Fame. She decides to randomly select two of her friends to go with her. Draw a tree diagram to illustrate the sample space, then list the resulting possibilities. (6 points)

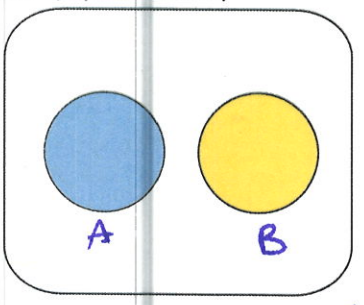


{BL, BJ, BR, LB, LJ, LR, JB, JL, JR, RB, RL, RJ}

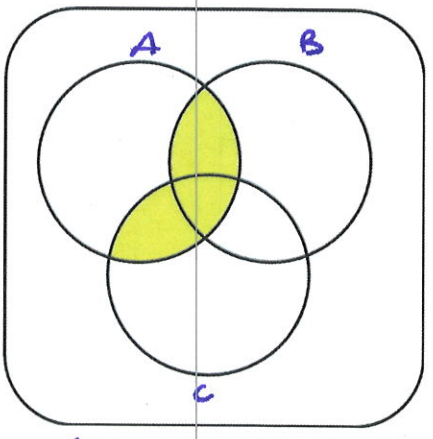
3. Venn diagrams are shown below illustrating sets and set concepts. Use set notation to describe each set. Label each set (A, B, etc.) in each diagram. (3 points each)



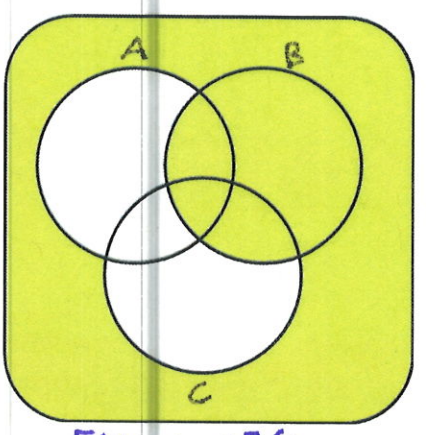
$A - B$ or $A \cap B'$



A & B are disjoint

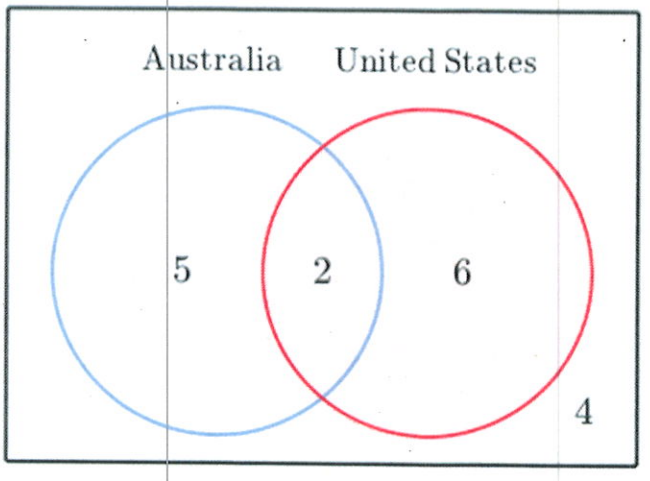


$(A \cap B) \cup (A \cap C)$



$[(A \cup C) - B]^c$

4. Convert the following Venn diagram into a two-way table if it illustrates the number of soccer fans in a sample that cheer for teams from the US or Australia. (5 points)



	fan of Australia	not a fan of Australia
Fan of US	2	6
Not a fan of US	5	4

5. Use the two-way table below to answer the questions that follow. (3 points each)

Gender	Preferred Program			
	Dance	Sports	Movies	Total
Women	16	6	8	30
Men	2	10	8	20
Total	18	16	16	50

a. What is the probability that a randomly selected person from this sample is a woman?

$\frac{30}{50} = \frac{3}{5} = 60\%$

b. What is the probability that a randomly selected person from this sample prefers to watch sports?

$\frac{16}{50} = \frac{8}{25} = 32\%$

c. What is the probability that a randomly selected person from this sample is both a woman and prefers to watch sports?

$\frac{6}{50} = \frac{3}{25} = 12\%$

d. What is the probability that a randomly selected person from this sample is either a woman or prefers to watch sports?

$\frac{30}{50} + \frac{16}{50} - \frac{6}{50} = \frac{40}{50} = \frac{4}{5} = 80\%$

e. What is the probability that a randomly selected person from this sample is a woman given that they watch sports?

$\frac{6}{16} = \frac{3}{8} = 37.5\%$

f. Are gender and preferred program independent? Why or why not? Show mathematical calculations to justify your answer.

$P(W) \cdot P(S)$ $\qquad$ $P(W \text{ and } S)$

$\frac{3}{5} \cdot \frac{8}{25} = \frac{24}{125} = 19.2\% \neq 12\%$

they are dependent

6. Find the value of the following expressions. (2 points each)

a. $5!$ b. $\frac{9!}{4!}$ c. $7P3$ d. $21C11$ e. $\binom{18}{7}$

120 15,120 352,716 31,824

210

7. Find the number of items in the set described by each scenario. (4 points each)

a. How many 5-card poker hands are there drawn from a standard deck of cards?

$52C5 = 2,598,960$

b. How many ways can we rearrange the letters of SUPERCALIFRAGILISTICEXPIALIDOCIOUS to form a puzzle?

$$\frac{34!}{2!\,2!\,2!\,2!\,2!\,3!\,3!\,3!\,6!\,2!} = 2.966 \times 10^{31}$$

S U P E R C A L I O

c. At Baskin Robbins, you decide to get three scoops of ice cream selected from their 31 possible flavors. If you don't select any flavors more than once, how many different kinds of "three-scoop" cones can you get?

$31P3 = 26,970$

d. You visit a restaurant having an á la carte special. You can select one item from each category for one special price. There are 3 appetizers to choose from, 5 main dishes, 2 sides, 4 desserts and 6 drinks. How many different meals are possible?

$3 \cdot 5 \cdot 2 \cdot 4 \cdot 6 = 720$

8. A committee is to consist of 8 faculty selected from a staff of 28. 18 of the faculty are women, and 10 are men. (4 points each)

a. What is the probability that the entire committee will be women?

$$\frac{\binom{18}{8}}{\binom{28}{8}} = \frac{43,758}{3,108,105} = 1.4\%$$

b. What is the probability that the committee will consist of exactly 4 women and 4 men?

$$\frac{\binom{18}{4}\binom{10}{4}}{\binom{28}{8}} = 20.7\%$$

c. What is the probability that the committee will have at least two men?

no men one man

$$\frac{\binom{10}{0}\binom{18}{8} + \binom{10}{1}\binom{18}{7}}{\binom{28}{8}} = 11.6\% \qquad 1 - .116 = 88.4\%$$

9. A variable is binomial with $n = 20, p = 0.30$. Find the following. (3 points each)

a. $P(x = 10)$

binomialpdf(20, .3, 10) = 0.0308

b. $P(x < 3)$

binomialcdf(20, .3, 2) = 0.03548 (3 not included)

c. $P(x \geq 16)$

1 − binomialcdf(20, .3, 15) =

5.55×10^{-6}

10. A charity holds a raffle and sells 1600 tickets for $2.00 each. They are awarding a first place prize of $1000, a second place prize of $500, and two third place prizes of $100. Find the expected value of purchasing a ticket. Interpret the result in the context of the problem. [Hint: it may help to construct a table.] (6 points)

$$998 * \frac{1}{1600} + 498 * \frac{1}{1600} + 98 * \frac{2}{1600} - 2 * \frac{1596}{1600} = -0.9375$$

For each ticket purchased, one can expect to lose $.9375. (around 94 cents)

11. In a standard deck of cards, 12 cards are face cards. Assume that one card is drawn at a time, recorded and replaced. What is the probability that if 5 cards are drawn, that 3 will be face cards? (6 points)

$p = \frac{12}{52}$ $n = 5$ $X = 3$

binomialpdf(5, 12/52, 3) = 7.3%

12. Is the distribution shown below a legitimate probability distribution? Why or why not? (4 points)

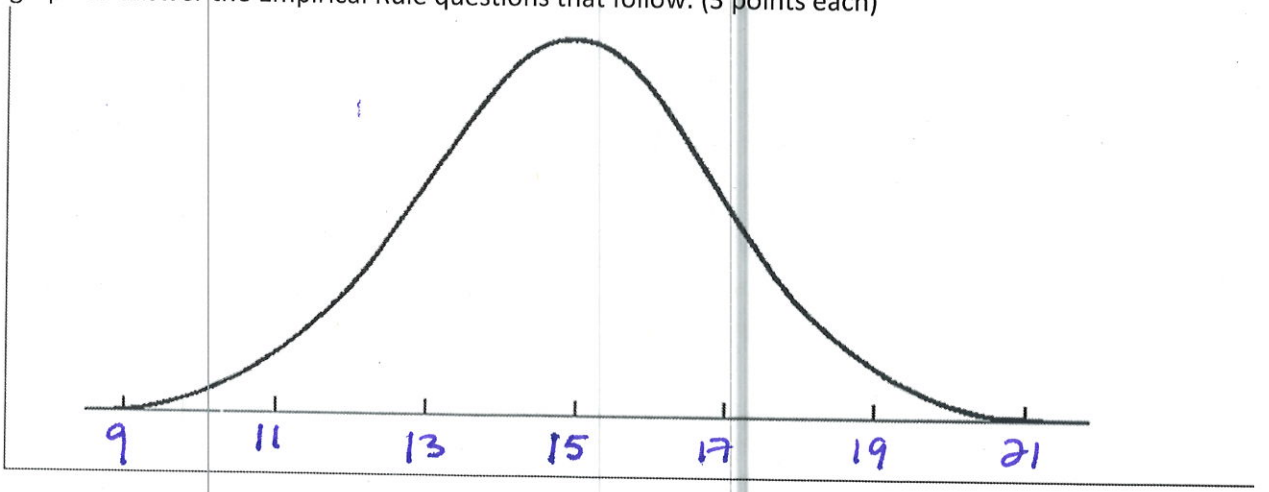
x	1	2	3	10	20	100
$P(x)$	0.1	$\frac{1}{5}$	−0.15	$\frac{1}{2}$	0.3	0.05

no, it is not probabilities cannot be negative

13. Fill in the missing value in the table below to make it a legitimate probability distribution. (3 points)

x	0	1	2	3	4	5 +
$P(x)$	0	0.5	0.25	0.125	0.0625	.0625

14. On the normal curve below, draw a normal distribution with mean of 15 and $\sigma = 2$. Use the graph to answer the Empirical Rule questions that follow. (3 points each)



a. What is the probability that a value will fall between 13 and 17? (3 points)

68%

b. What is the probability that a value will fall between 11 and 17? (3 points)

$\frac{68+95}{2} = 81.5\%$

c. What is the probability that a value will be greater than 19? (3 points)

2.5%

d. What is the percentile of the value 13? (3 points)

16%

e. What is the z-score of the value 21.8? (3 points)

$\frac{21.8-15}{2} = 3.4$

15. Find the probabilities associated with a standard normal curve. (3 points each)

a. $P(z > 2.31)$

normalcdf(2.31, E99) = .0104

b. $P(z < -1.05)$

normalcdf(-E99, -1.05) = .1469

c. $P(-1.9 < z < 0.75)$

normalcdf(-1.9, 0.75) = .7447

d. $P(z < -1.26, or\ z > 1.26)$

2* normalcdf(-E99, -1.26) = .2077

16. IQ is normally distributed with a mean of 100 and a standard deviation of 15. What is the probability of having an IQ greater than 93? (4 points)

normalcdf(93, E99, 100, 15) = .6796

17. Given the probabilities stated below, find the z-score they correspond to in a standard normal distribution. (3 points each)

a. $P(z < a) = 0.8761, a = ?$

invNorm(.8761) = 1.1557

b. $P(z > a) = 0.3455, a = ?$

invNorm(.6555) = .4002

c. $P(-a < z < a) = 0.75, a = ?$

1 - .75 = .25 invNorm(.125) = -1.1503

$\pm$ 1.1503

d. $P(z < -a, or\ z > a) = 0.02, a = ?$

invNorm(.01) = -2.3263

and 2.3263

18. If the ACT has a mean of 21.6 and a standard deviation of 5.2, above what score does the top 10% of scores fall? (4 points)

invNorm(.90, 21.6, 5.2) = 28.26

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