MAT 212, Quiz #7, Spring 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Be sure to answer all parts of each question.

1. Determine if the vectors $\begin{bmatrix}4\\1\\3\end{bmatrix}, \begin{bmatrix}6\\0\\2\end{bmatrix}, \begin{bmatrix}1\\3\\1\end{bmatrix}$ are independent.

$\det\begin{bmatrix}4&6&1\\1&0&3\\3&2&1\end{bmatrix} = 26 \neq 0$ independent

(row-reduces to identity)

2. Determine if the solutions $y_1 = \cos 2x, y_2 = \sin 2x, y_3 = e^{-3x}$ form a fundamental solution set.

$$\begin{vmatrix}\cos 2x & \sin 2x & e^{-3x}\\ -2\sin 2x & 2\cos 2x & -3e^{-3x}\\ -4\cos 2x & -4\sin 2x & 9e^{-3x}\end{vmatrix}$$

$$\cos 2x \begin{vmatrix}2\cos 2x & -3e^{-3x}\\ -4\sin 2x & 9e^{-3x}\end{vmatrix} - \sin 2x \begin{vmatrix}-2\sin 2x & -3e^{-3x}\\ -4\cos 2x & 9e^{-3x}\end{vmatrix} + e^{-3x}\begin{vmatrix}-2\sin 2x & 2\cos 2x\\ -4\cos 2x & -4\sin 2x\end{vmatrix}$$

$18\cos^2 2x \cdot e^{-3x} - 12e^{-3x}\cos 2x \sin 2x + 18\sin^2 2x\, e^{-3x} + 12e^{-3x}\cos 2x \sin 2x + e^{-3x}(8\sin^2 2x + 8\cos^2 2x)$

$18e^{-3x} + 8e^{-3x} = 26e^{-3x}$ yes $\neq 0$, a fundamental set.

continues ↓

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2. Determine if the solutions $y_1 = \cos 2x, y_2 = \sin 2x, y_3 = e^{-3x}$ form a fundamental solution set.

3. A mass weighing 100 lbs. stretches a spring 8 inches. The spring is also attached to a damping mechanism that imposes a damping force numerically twice that of the velocity. Set up a differential equation that models the system. The mass is pulled down 4 inches from equilibrium and released with a downward velocity of 3 inches/sec. If the system is unforced, solve the system. Round your answers to 4 decimal places.

$m = \frac{100}{32} = \frac{25}{8}$ slugs

$100 = k(\frac{2}{3})$

$k = 150$

$\frac{25}{8}y'' + 2y' + 150y = 0$

$y'' + \frac{16}{25}y' + 48y = 0$

$y(0) = -1/3$

$y'(0) = -1/4$

$.64$

$r = -.64 \pm \frac{\sqrt{.4096 - 192}}{2} = -.32 \pm 6.9208i$

$y = c_1 e^{-.32t}\cos(6.9208t) + c_2 e^{-.32t}\sin(6.9208t)$

$y(0) = -1/3 = c_1 \Rightarrow c_1 = -1/3$

$y'(t) = -.32c_1 e^{-.32t}\cos(6.9208t) + -6.9208c_1 e^{-.32t}\sin(6.9208t) + -.32e^{-.32t}c_2\sin(6.9208t) + 6.9208e^{-.32t}\cos(6.9208t)c_2$

$y'(0) = -.32\underset{(-1/3)}{c_1} + 6.9208c_2 = -1/4$

$c_2 = -.0515$

$y(t) = -1/3e^{-.32t}\cos(6.9208t) - .0515e^{-.32t}\sin(6.9208t)$