MAT 212, Quiz #4, Spring 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Be sure to answer all parts of each question.

1. Find the determinant of $\begin{bmatrix} 1 & 3 & 3 \\ -7 & 0 & -4 \\ 3 & -2 & 5 \end{bmatrix}$.

$$1\begin{vmatrix} 0 & -4 \\ -2 & 5 \end{vmatrix} - 3\begin{vmatrix} -7 & -4 \\ 3 & 5 \end{vmatrix} + 3\begin{vmatrix} -7 & 0 \\ 3 & -2 \end{vmatrix}$$

$$1(0-8) - 3(-35+12) + 3(14-0)$$

$$-8 + 69 + 42 = 103$$

2. A tank contains 1000 L of a solution of 100 kg of salt dissolved in the water. Pure water is pumped into the tank at a rate of 5L/s and the well-stirred mixture is pumped out at the same rate. How long will it be before 10 kg of salt remains?

$\frac{dA}{dt} = (0)\left(\frac{5L}{s}\right) - \frac{A}{1000L}\left(\frac{5L}{s}\right) \Rightarrow \frac{dA}{dt} = \frac{-A}{200}$ $\quad A(0) = 100\ kg.$

$\int \frac{dA}{A} = \int \frac{-1}{200}\,dt$

$\ln A = \frac{-1}{200}t + C$

$A = e^{\frac{-1}{200}t + C} \Rightarrow A(t) = A_0 e^{-1/200\,t}$ $\quad A_0 = 100$

$A(t) = 100e^{-1/200\,t}$

$t = 460.5$ seconds

3. Use Euler's Method to approximate the value of $y(1)$ for $y' = x + \sqrt{y}$ if $y(0) = 1$ using 5 steps.

$\Delta x = 1/5 = .2$

$x_0 = 0,\ y_0 = 1 \quad m_0 = 0 + \sqrt{1} = 1 \quad y_1 = 1(.2) + 1 = 1.2$

$x_1 = .2 \quad y_1 = 1.2 \quad m_1 = .2 + \sqrt{1.2} = 1.295 \quad y_2 = 1.295(.2) + 1.2 = 1.459$

$x_2 = .4 \quad y_2 = 1.459 \quad m_2 = .4 + \sqrt{1.459} = 1.6079 \quad y_3 = 1.6079(.2) + 1.459 = 1.781$

$x_3 = .6,\ y_3 = 1.781 \quad m_3 = .6 + \sqrt{1.781} = 1.934 \quad y_4 = 1.934(.2) + 1.781 = 2.1679$

$x_4 = .8,\ y_4 = 2.1679 \quad m_4 = .8 + \sqrt{2.1674} = 2.272 \quad y_5 = 2.272(.2) + 2.1679 = 2.622$

$x_5 = 1 \quad y_5 = 2.622$

$y(1) \approx 2.622$