MAT 142, Quiz #11, Spring 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round.

1. Verify the identities.
   a. $\sec^2 x \csc^2 x = \sec^2 x + \csc^2 x$

$(1+\tan^2 x)(1+\cot^2 x) =$

$1+\tan^2 x+\cot^2 x+1 = \sec^2 x+\csc^2 x$

   b. $\tan^2 x + \sin^2 x + \cos^2 x = \sec^2 x$

$\tan^2 x + 1 = \sec^2 x$

   c. $\tan\left(\frac{\pi}{4}-\theta\right) = \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

$$\frac{\tan \pi/4 - \tan\theta}{1+\tan\frac{\pi}{4}\cdot\tan\theta} = \frac{1-\tan\theta}{1+\tan\theta}\cdot\frac{\cos\theta}{\cos\theta}$$

$$= \frac{\cos\theta - \frac{\sin\theta}{\cos\theta}\cdot\cos\theta}{\cos\theta + \frac{\sin\theta}{\cos\theta}\cdot\cos\theta} = \frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

2. Use the sum and difference formulas to find the exact value of:

a. $\frac{\tan 10° + \tan 35°}{1 - \tan 10° \tan 35°}$

$= \tan(10° + 35°) = \tan 45° = 1$

b. $\cos 285°$ $= \cos(240° + 45°) =$

$\cos 240° \cos 45° - \sin 240° \sin 45°$

$\left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{-1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

Some useful formulas:

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$

$$\sin(a-b) = \sin a \cos b - \sin b \cos a$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$