MAT 142 Homework #8 Key

1a. $\tan x \csc x \cos x = 1$ (prove)

$$\frac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \frac{1}{\cancel{\sin x}} \cdot \cancel{\cos x} = 1$$

b. $$\frac{\cos\theta \sec\theta}{\cot\theta} = \frac{\cancel{\cos\theta} \cdot \frac{1}{\cancel{\cos\theta}}}{\cot\theta} = \frac{1}{\cot\theta} = \tan\theta$$

c. $$\frac{\sin t}{\tan t} + \frac{\cos t}{\cot t} = \frac{\sin t}{\left(\frac{\sin t}{\cos t}\right)} + \frac{\cos t}{\left(\frac{\cos t}{\sin t}\right)} = \cancel{\sin t}\left(\frac{\cos t}{\cancel{\sin t}}\right) + \cancel{\cos t}\left(\frac{\sin t}{\cancel{\cos t}}\right)$$

$$= \cos t + \sin t$$

d. $$\frac{\sec x - \csc x}{\sec x + \csc x} \cdot \frac{\sin x + \cos x}{\sin x + \cos x} = \frac{\frac{\sin x}{\cos x} + 1 - 1 - \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + 1 + 1 + \frac{\cos x}{\sin x}} =$$

$$\frac{\tan x - \cot x}{\tan x + 2 + \cot x} \cdot \frac{\tan x}{\tan x} = \frac{\tan^2 x - 1}{\tan^2 x + 2\tan x + 1} = \frac{\cancel{(\tan x + 1)}(\tan x - 1)}{\cancel{(\tan x + 1)}(\tan x + 1)}$$

$$= \frac{\tan x - 1}{\tan x + 1}$$

e. $$\frac{1 + \cos t}{1 - \cos t} \frac{\frac{1}{\sin t}}{\frac{1}{\sin t}} = \frac{\csc t + \cot t}{\csc t - \cot x} \cdot \frac{\csc t + \cot t}{\csc t + \cot t} = \frac{(\csc t + \cot t)^2}{\underbrace{\csc^2 t - \cot^2 t}_{=1}}$$

$$= (\csc t + \cot t)^2$$

f. $$(\tan^2\theta + 1)(\cos^2 t + 1) = \frac{\sin^2 t}{\cancel{\cos^2 t}} \cancel{\cos^2 t} + \tan^2 t + \cos^2 t + 1 =$$

$$\sin^2 t + \cos^2 t + \tan^2 t + 1 = 1 + \tan^2 t + 1 = \tan^2 t + 2$$

1g. $\dfrac{\cos(x+h) - \cos x}{h} = \dfrac{\cos x \cos h - \sin x \sin h - \cos x}{h}$

$\cos x \left(\dfrac{\cos h - 1}{h}\right) - \sin x \left(\dfrac{\sin h}{h}\right)$

h. $(\sin\theta + \cos\theta)^2 = \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta =$

$1 + 2\sin\theta\cos\theta = 1 + \sin 2\theta$

i. $\sin 4t = 2\sin 2t \cos 2t = 2(2\sin t\cos t)(\cos^2 t - \sin^2 t)$

$= 4\sin t\cos^3 t - 4\sin^3 t\cos t$

j. $\tan\left(\frac{x}{2}\right) + \cot\frac{x}{2} = \dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x} = \dfrac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1+\cos x)}$

$= \dfrac{2 + 2\cos x}{\sin x(1+\cos x)} = \dfrac{2\cancel{(1+\cos x)}}{\sin x\cancel{(1+\cos x)}} = \dfrac{2}{\sin x} = 2\csc x$

k. $\cos^2 x - \sin^2 x = \cos^2 x - (1-\cos^2 x) = \cos^2 x + \cos^2 x - 1 =$

$2\cos^2 x - 1$

l. $\sin t \tan t = \sin t\,\dfrac{\sin t}{\cos t} = \dfrac{\sin^2 t}{\cos t} = \dfrac{1-\cos^2 t}{\cos t}$

m. $1 - \dfrac{\cos^2 x}{1+\sin x} = 1 - \dfrac{(1-\sin^2 x)}{1+\sin x} = 1 - \dfrac{\cancel{(1+\sin x)}(1-\sin x)}{\cancel{1+\sin x}} =$

$1 - (1-\sin x) = \cancel{1} - \cancel{1} + \sin x = \sin x.$

n. $\dfrac{\tan x + \tan y}{1 - \tan x\tan y} = \tan(x+y) = \dfrac{\sin(x+y)}{\cos(x+y)} = \dfrac{\sin x\cos y + \cos x\sin y}{\cos x\cos y - \sin x\sin y}$

o. $\dfrac{\cos^2 t + 4\cos t + 4}{\cos t + 2} = \dfrac{(\cos t + 2)^{\cancel{2}}}{\cancel{\cos t + 2}} = \cos t + 2 \cdot \dfrac{\sec t}{\sec t} = \dfrac{1 + 2\sec t}{\sec t}$

1p. $$\frac{\sin x - \cos x + 1}{\sin x + \cos x - 1} = \frac{(\sin x + 1) - \cos x}{\sin x + \cos x - 1} \cdot \frac{(\sin x + 1) + \cos x}{\sin x + \cos x + 1}$$

$$= \frac{(\sin x + 1)^2 - \cos^2 x}{\underbrace{\sin^2 x + \cos x \sin x + \cancel{\sin x} + \cos x \sin x + \cos^2 x}_{\cancel{=1}} + \cancel{\cos x} - \cancel{\sin x} - \cancel{\cos x} - \cancel{1}}$$

$$= \frac{(\sin x + 1)^2 - \cos^2 x}{2 \cos x \sin x} = \frac{\sin^2 x + 2 \sin x + 1 - (1 - \sin^2 x)}{2 \cos x \sin x} =$$

$$\frac{\sin^2 x + 2 \sin x + \cancel{1} - \cancel{1} + \sin^2 x}{2 \cos x \sin x} = \frac{2 \sin^2 x + 2 \sin x}{2 \cos x \sin x} =$$

$$\frac{\cancel{2} \cancel{\sin x} (\sin x + 1)}{\cancel{2} \cos x \cancel{\sin x}} = \frac{\sin x + 1}{\cos x}$$

q. $$\frac{\sin(\alpha - \beta)}{\cos \alpha \cos \beta} = \frac{\sin \alpha \cos \beta - \cos \alpha \sin \beta}{\cos \alpha \cos \beta} = \frac{\sin \alpha \cancel{\cos \beta}}{\cos \alpha \cancel{\cos \beta}} - \frac{\cancel{\cos \alpha} \sin \beta}{\cancel{\cos \alpha} \cos \beta}$$

$$= \tan \alpha - \tan \beta$$

r. $$1 - \tan^2 x = 1 - \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x - \sin^2 x}{\cos^2 x} = \frac{\cos 2x}{\cos^2 x}$$

s. $$2 \tan \frac{\alpha}{2} = 2\left[\frac{\sin \alpha}{1 + \cos \alpha}\right] \cdot \frac{\sin \alpha}{\sin \alpha} = \frac{2 \sin^2 \alpha}{\sin \alpha (1 + \cos \alpha)} = \frac{\sin^2 \alpha + \sin^2 \alpha}{\sin \alpha (1 + \cos \alpha)}$$

$$= \frac{\sin^2 \alpha + 1 - \cos^2 \alpha}{\sin \alpha (1 + \cos \alpha)}$$

2a. $$\cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right) = \cos(3\pi/4) \cos(\pi/6) + \sin(3\pi/4) \sin(\pi/6) =$$

$$-\frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{1 - \sqrt{3}}{2\sqrt{2}}$$

2b. $\cos 75° = \cos(30°+45°) = \cos 30° \cos 45° - \sin 30° \sin 45°$

$= \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}}\right) - \frac{1}{2}\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}-1}{2\sqrt{2}}$

c. $\frac{\tan\frac{\pi}{5} - \tan\frac{\pi}{30}}{1 + \tan\frac{\pi}{5}\tan\frac{\pi}{30}} = \tan\left(\frac{\pi}{5} - \frac{\pi}{30}\right) = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

d. $\frac{2\tan \pi/8}{1+\tan^2(\pi/8)} = \tan \pi/4 = 1$

e. $\tan(3\pi/8) = \frac{\sin 3\pi/4}{1+\cos 3\pi/4} = \frac{1/\sqrt{2}}{1+{}^{-1}/\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}-1}$

f. $\cos\left(\frac{5\pi}{18}\right)\cos(\pi/9) + \sin(5\pi/18)\sin(\pi/9) = \cos\left(\frac{5\pi}{18} - \pi/9\right) = \cos(\pi/6) = \frac{\sqrt{3}}{2}$

g. $\tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right) = \frac{\tan\pi/6 + \tan\pi/4}{1 - \tan\pi/6 \tan\pi/4} = \frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}(1)} \frac{\sqrt{3}}{\sqrt{3}} = \frac{1+\sqrt{3}}{\sqrt{3}-1}$

h. $2\sin 15° \cos 15° = \sin 30° = \frac{1}{2}$

i. $\sin 105° = \sin(60°+45°) = \sin 60° \cos 45° + \cos 60° \sin 45° =$

$\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} + \frac{1}{2}\frac{1}{\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

3. $\sin(\cos^{-1}\frac{1}{2})\cos(\sin^{-1}\frac{3}{5}) + \cos(\cos^{-1}\frac{1}{2})\sin(\sin^{-1}\frac{3}{5}) =$

$\frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{5} + \frac{1}{2} \cdot \frac{3}{5} = \frac{4\sqrt{3}+3}{10}$

b. $2\sin(\sin^{-1}\frac{\sqrt{3}}{2})\cos(\sin^{-1}\frac{\sqrt{3}}{2}) = \cancel{2}\frac{\sqrt{3}}{2} \cdot \frac{1}{\cancel{2}} = \frac{\sqrt{3}}{2}$

c. $\cos(\tan^{-1}\frac{4}{3})\cos(\cos^{-1}\frac{5}{13}) - \sin(\tan^{-1}\frac{4}{3})\sin(\cos^{-1}\frac{5}{13})$

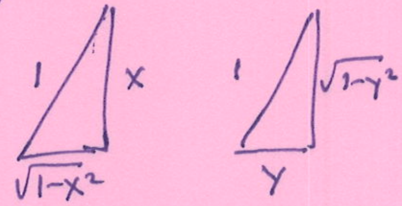
$= \frac{3}{\cancel{5}} \cdot \frac{\cancel{5}}{13} - \frac{4}{5} \cdot \frac{12}{13} = -\frac{33}{65}$

d. $\cos^2\left(\frac{1}{2}\sin^{-1}\frac{3}{5}\right) = \frac{1+\cos(\sin^{-1}\frac{3}{5})}{2} = \frac{1+\frac{4}{5}}{2} = \frac{\frac{9}{5}}{2} = \frac{9}{10}$

4a.

$\cos(\sin^{-1}x - \cos^{-1}y) =$

$\cos(\sin^{-1}x)\cos(\cos^{-1}y) + \sin(\sin^{-1}x)\sin(\cos^{-1}y)$

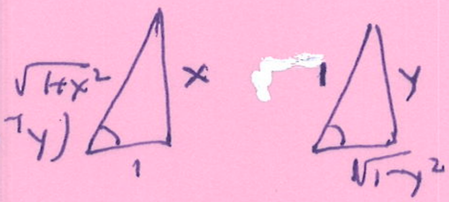


$\sqrt{1-x^2} \cdot y + x\sqrt{1-y^2} =$

$y\sqrt{1-x^2} + x\sqrt{1-y^2}$

b. $\sin(\tan^{-1}x - \sin^{-1}y)$

$\sin(\tan^{-1}x)\cos(\sin^{-1}y) - \cos(\tan^{-1}x)\sin(\sin^{-1}y)$

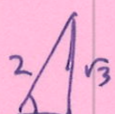


$\frac{x}{\sqrt{1+x^2}} \cdot \frac{\sqrt{1-y^2}}{1} - \frac{1}{\sqrt{1+x^2}} \cdot y$

$= \frac{x\sqrt{1-y^2} - y}{\sqrt{1+x^2}}$

5. $\sin x = \frac{\sqrt{3}}{2}$

a.



$x = \frac{\pi}{3}, \frac{2\pi}{3}$

b. $\tan\frac{x}{2} = \sqrt{3}$ $\qquad \alpha = \frac{x}{2}$

$\tan\alpha = \sqrt{3}$

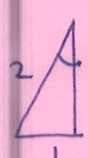
$\alpha = \frac{\pi}{3} \Rightarrow \frac{x}{2} = \frac{\pi}{3} \Rightarrow x = \frac{2\pi}{3}$

$\frac{x}{2} = \frac{2\pi}{3} \Rightarrow x = \frac{4\pi}{3}$

c. $2\sin^2 x - \sin x - 1 = 0$

$2u^2 - u - 1 = 0$

$(2u+1)(u-1) = 0$



$\sin x = -\frac{1}{2}$ $\qquad \sin x = 1$ $\qquad x = \pi/2$

$x = 7\pi/6, 11\pi/6$

5d. $\sec^2 x - 2 = 0$

$\sec^2 x = 2$

$\sec x = \pm\sqrt{2}$

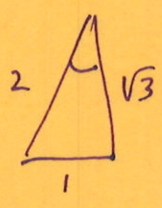
$x = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$

g. $2\cos x + \sqrt{3} = 0$

$2\cos x = -\sqrt{3}$

$\cos x = -\frac{\sqrt{3}}{2}$

$x = 5\pi/6, 7\pi/6$



h. $\sin(2x + \pi/6) = \frac{1}{2}$ $\alpha = 2x + \pi/6$

$\sin\alpha = 1/2$

$\alpha = \pi/6, 5\pi/6, 13\pi/6, 17\pi/6, 25\pi/6$

$2x + \pi/6 = \pi/6$

$2x = 0$

$x = 0$

$2x + \pi/6 = 5\pi/6$

$2x = 2\pi/3$

$x = \pi/3$

$2x + \pi/6 = 13\pi/6$

$2x = 2\pi$

$x = \pi$

$2x + \pi/6 = 17\pi/6$

$2x = 8\pi/3$

$x = 4\pi/3$

$2x + \pi/6 = 25\pi/6$

$2x = 24\pi/6 = 4\pi$

$x = 2\pi$

i. $\cos^2 x + 2\cos x - 3 = 0$

$u^2 + 2u - 3 = 0$

$(u+3)(u-1) = 0$

$\cos x = -3$ not possible

$\cos x = 1$ $x = 0, 2\pi$

j. $\sin x + 2\sin x \cos x = 0$

$\sin x(1 + 2\cos x) = 0$

$\sin x = 0$

$x = 0, \pi, 2\pi$

$1 + 2\cos x = 0$

$\cos x = -\frac{1}{2}$

$x = 2\pi/3, 4\pi/3$

5i. $\cos 2x = \cos x$

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$2u^2 - u - 1 = 0$

$(2u+1)(u-1) = 0$

$\cos x = -\frac{1}{2}$ $\quad$ $\cos x = 1$

$x = 2\pi/3,\ 4\pi/3$ $\quad$ $x = 0, 2\pi$

j. $5\sin x = 2\cos^2 x - 4$

$5\sin x = 2(1-\sin^2 x) - 4$

$5\sin x = 2 - 2\sin^2 x - 4$

$2\sin^2 x + 5\sin x + 2 = 0$

$(2\sin x + 1)(\sin x + 2) = 0$

$\sin x = -1/2$ $\quad$ $\sin x = -2$ not possible

$x = 7\pi/6,\ 11\pi/6$

k. $(\sin x + \cos x)^2 = (1)^2$

$\sin x^2 + 2\sin x\cos x + \cos^2 x = 1$

$\cancel{1} + 2\sin x\cos x = \cancel{1}$

$\sin 2x = 0$

$2x = 0, \pi, 2\pi, 3\pi, 4\pi$

$x = 0,\ \pi/2,\ \underset{=-1}{\cancel{\pi}},\ \underset{=-1}{\cancel{3\pi/2}},\ 2\pi$

l. $2\cos^3 x + \cos^2 x - 2\cos x - 1 = 0$

$2u^3 + u^2 - 2u - 1 = 0$

$u^2(2u+1) - 1(2u+1) = 0$

$(u^2-1)(2u+1) = (u-1)(u+1)(2u+1) = 0$

$\cos x = 1$ $\quad$ $\cos x = -1$ $\quad$ $2\cos x + 1 = 0$ $\quad$ $x = 2\pi/3,\ 4\pi/3$

$x = 0, 2\pi$ $\quad$ $x = \pi$ $\quad$ $\cos x = -1/2$

6\. a. $\cos x = -\frac{2}{5}$

$x = \cos^{-1}(-2/5) = 1.9823$ radians QII

and 4.3009 radians QIII

b. $4\tan^2 x - 8\tan x + 3 = 0$

$4u^2 - 8u + 3 = 0$

$(2u - 3)(2u - 1) = 0$

$\tan x = 3/2$, $\tan x = 1/2$

.98 radians .4636 radians

4.12 radians 3.605 radians

c. $5\sin^2 x - 1 = 0$

$\sin^2 x = 1/5$

$\sin x = \pm\sqrt{1/5}$

$x = .4636$ radians

2.6779 radians

3.605 radians

5.8195 radians

d. $2\sin 3x + \sqrt{3} = 0$

$2\sin 3x = -\sqrt{3}$

$\sin 3x = -\frac{\sqrt{3}}{2}$

$3x = 4\pi/3,\ 5\pi/3,\ 10\pi/3,\ 11\pi/3,\ 16\pi/3,\ 17\pi/3$

$x = 4\pi/9,\ 5\pi/9,\ 10\pi/9,\ 11\pi/9,\ 16\pi/9,\ 17\pi/9$