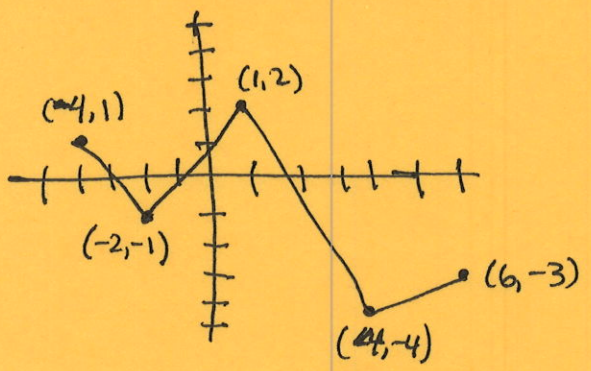
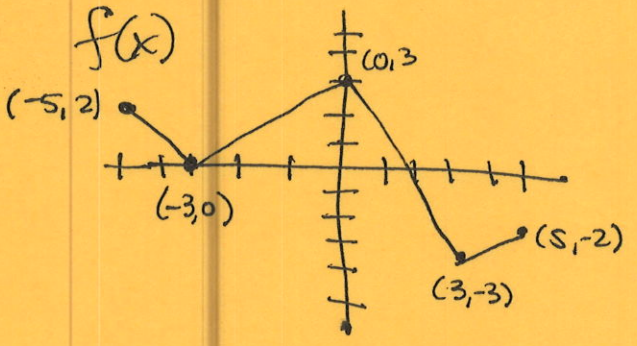
MAT 142 Homework #3 Key

1a. $f(x-1)-1$

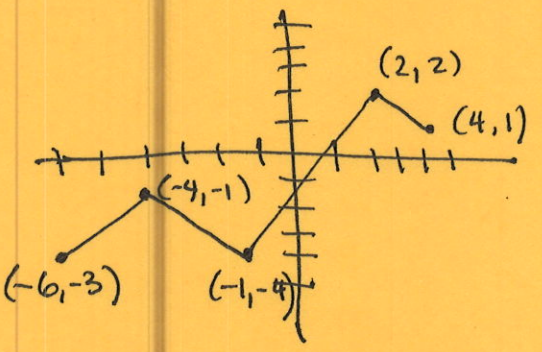
move +1 in x
move -1 in y





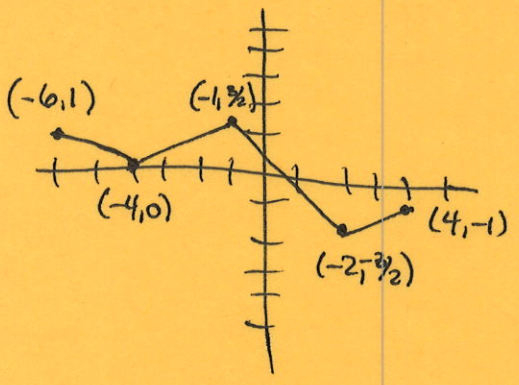
b. $-f(x+1)-1$

move -1 in y
reflect vertically (flip sign of y)
move -1 in y



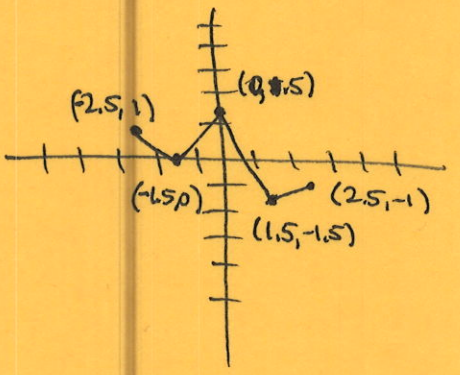
c. $\frac{1}{2}f(x+1)$

move left 1 in x
compress y by $\frac{1}{2}$



d. $\frac{1}{2}f(2x)$

Compress x by 2
Compress y by 1/2



2. See next page(s)

3. a. $f+g = 2x^2-x+3+x+1 = 2x^2+4$

D: {x | x is all reals}
R: $[4, \infty)$

b. $fg = (2x^2-x+3)(x+1) =$

$2x^3+2x^2-x^2-x+3x+3 = 2x^3+x^2+2x+3$

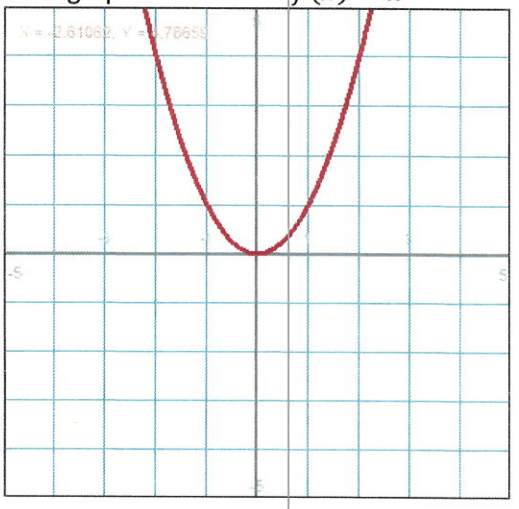
D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

c. $\frac{g}{f} = \frac{x+1}{} = \frac{x+1}{} = \frac{1}{2x+3}$

D: $x \neq -1, -\frac{3}{2}$ $y \neq 0, y \neq \frac{-1}{5}$

#2

Base graph $f(x) = x^2$



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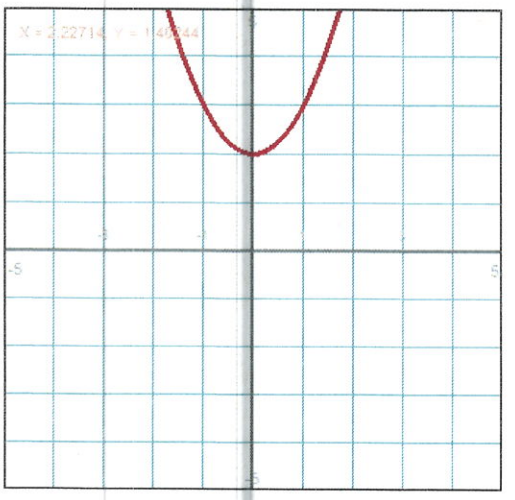

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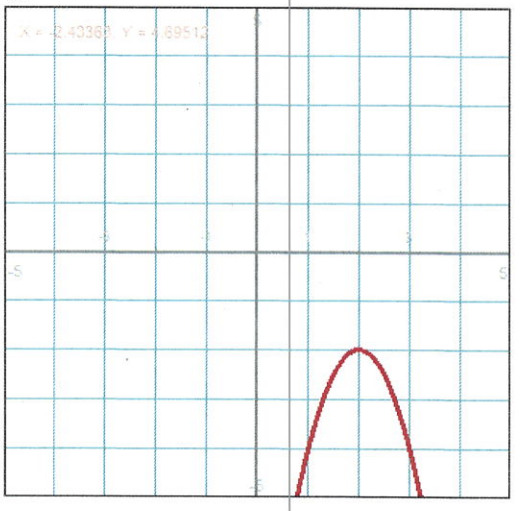

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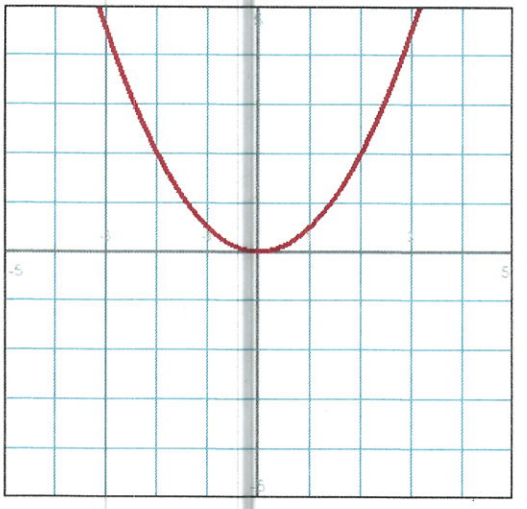

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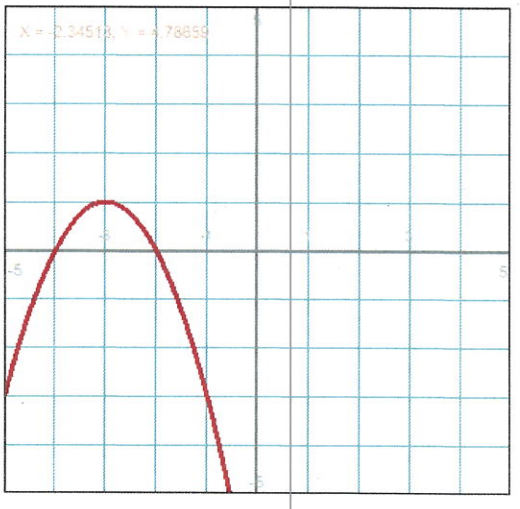

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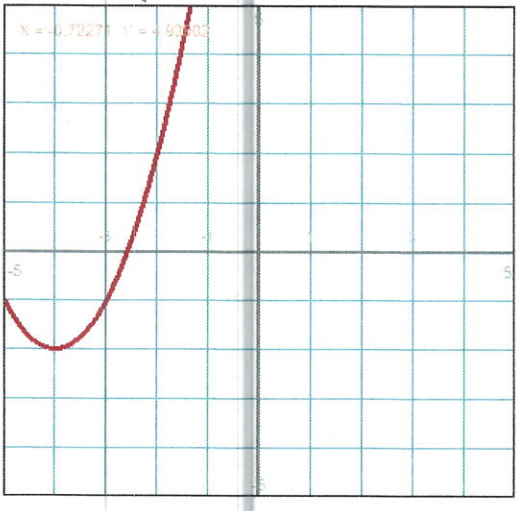

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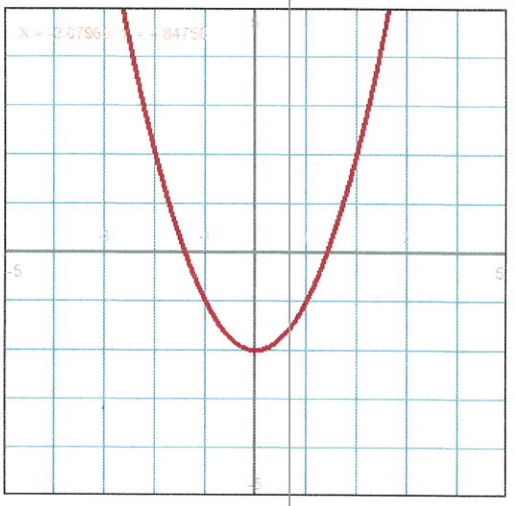
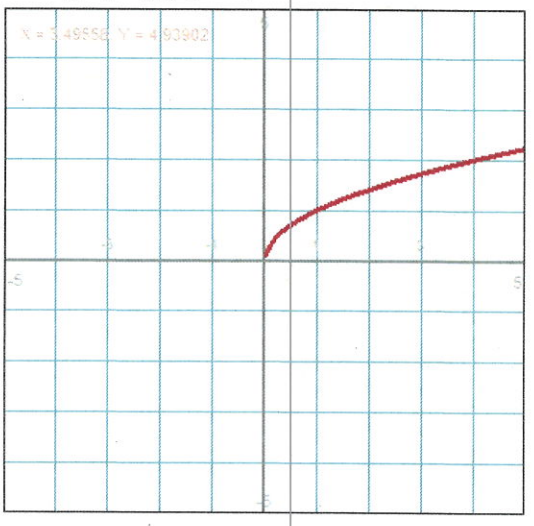


Base graph $g(x) = \sqrt{x}$



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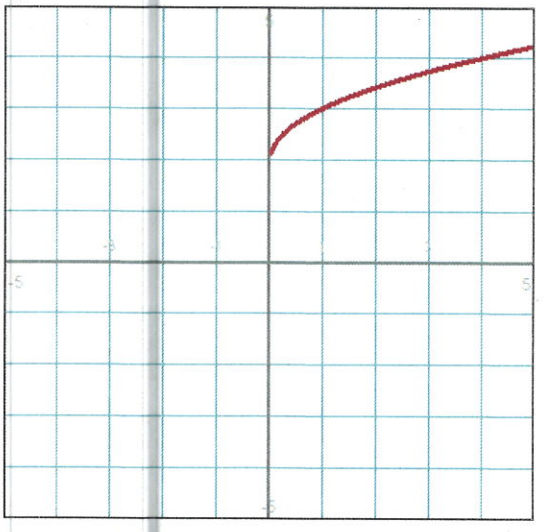

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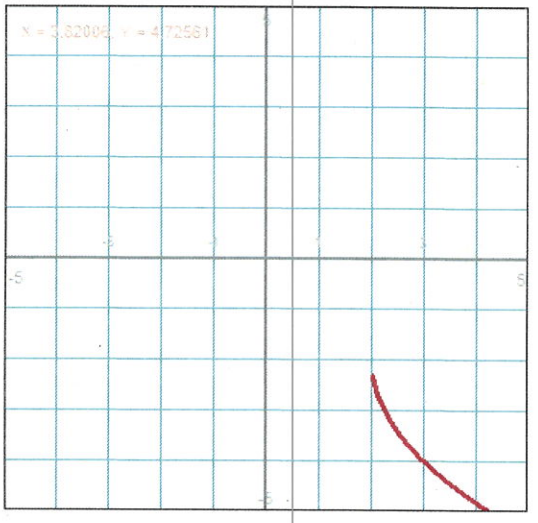

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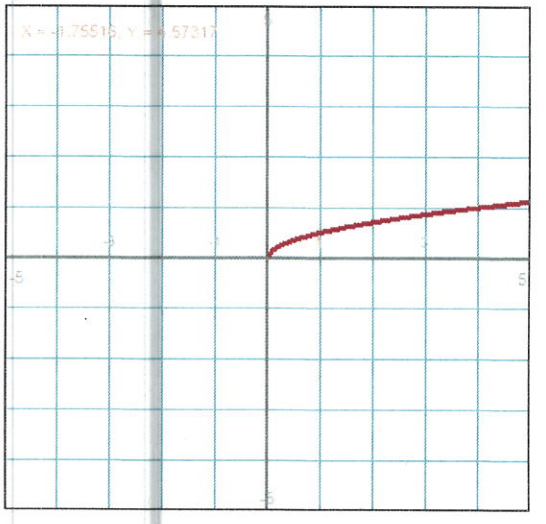

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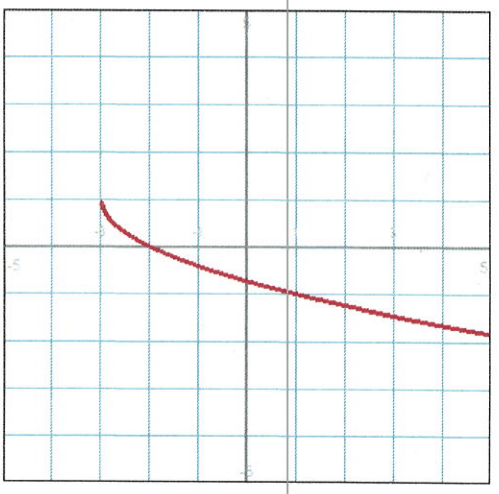

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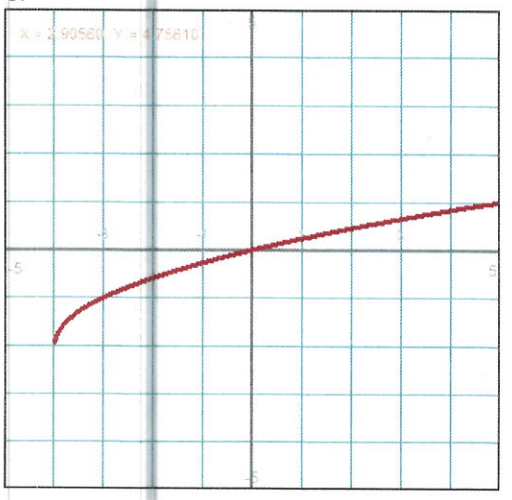

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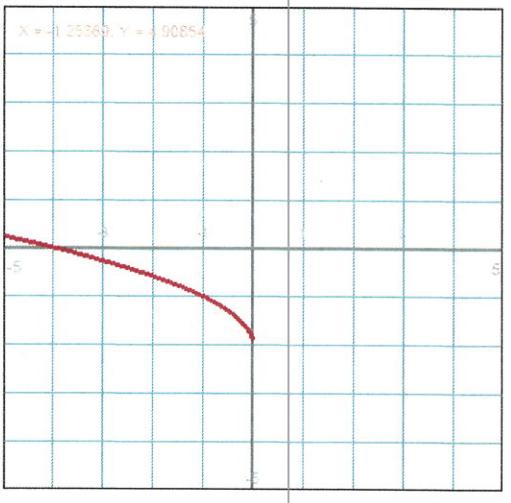
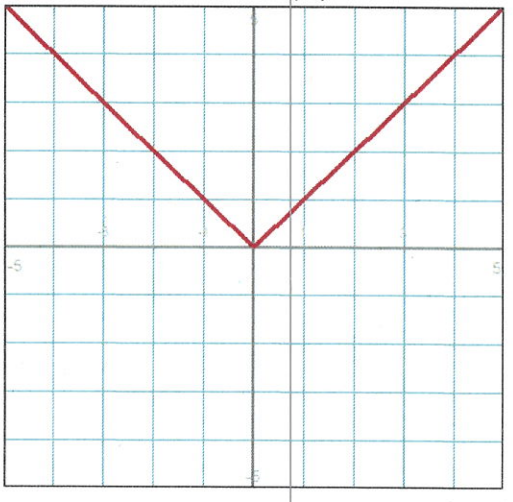


Base graph $h(x) = |x|$



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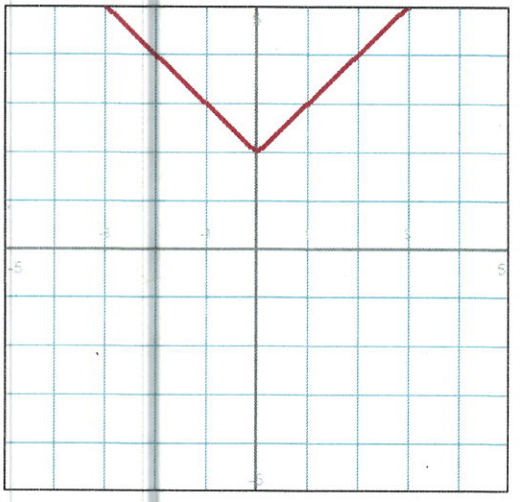

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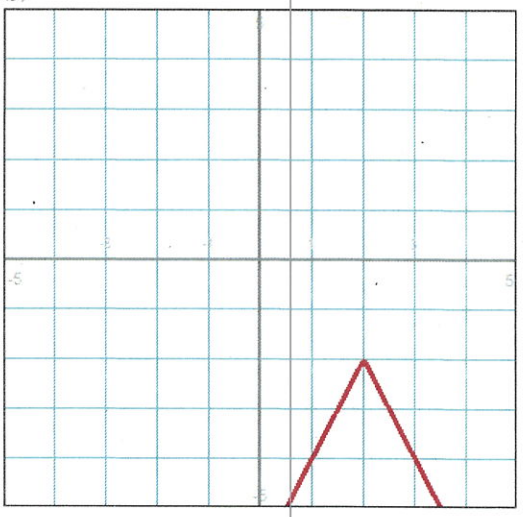

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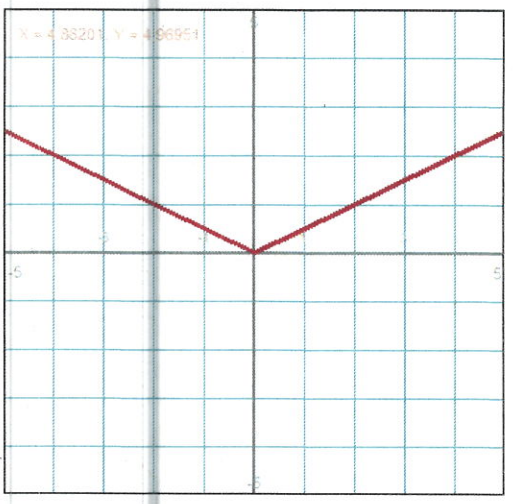

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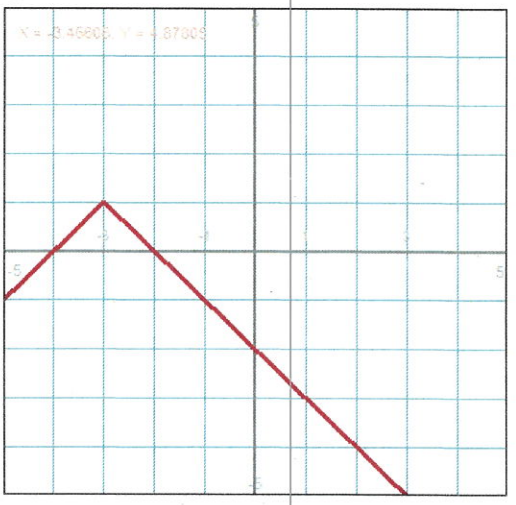

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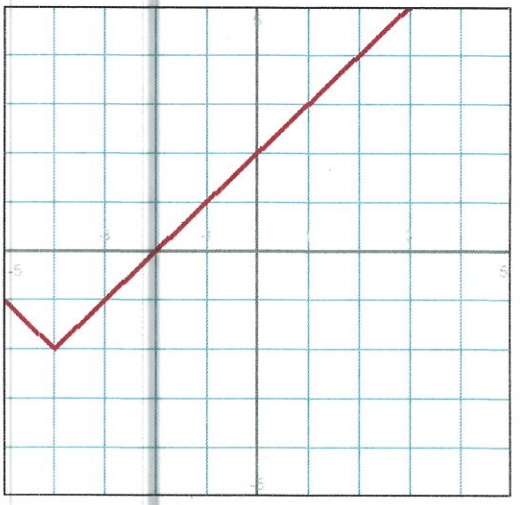

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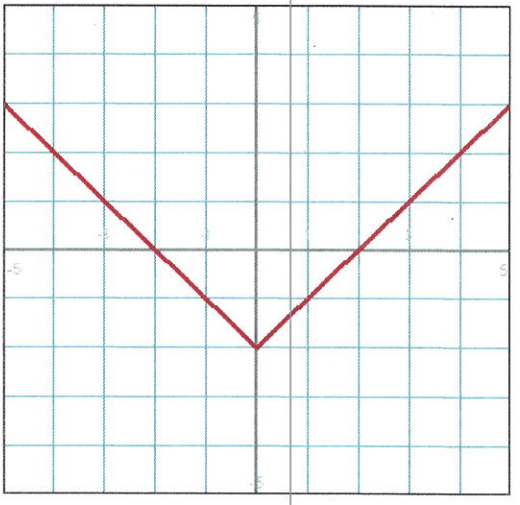
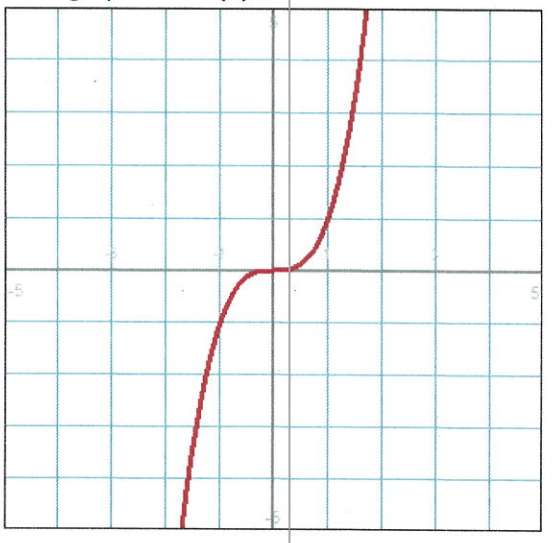


Base graph $k(x) = x^3$



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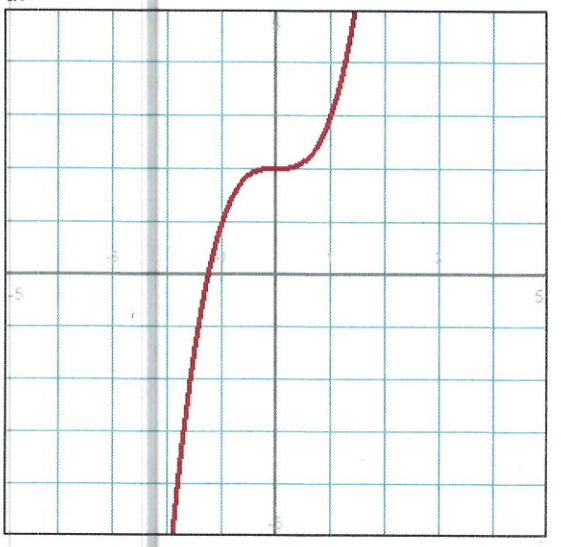

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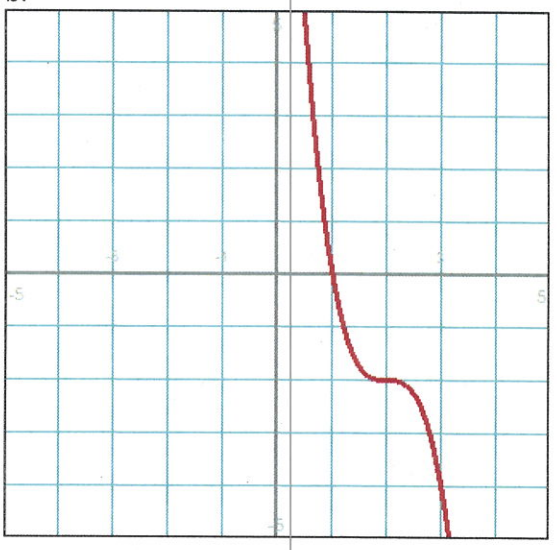

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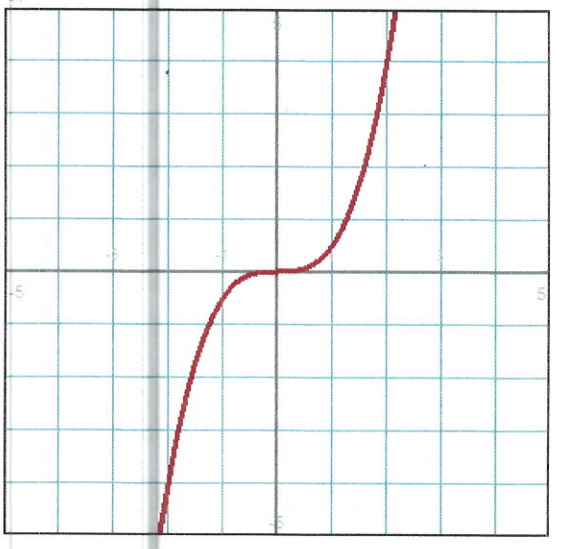

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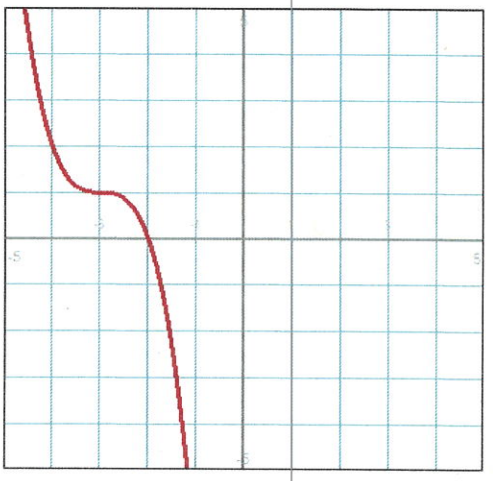

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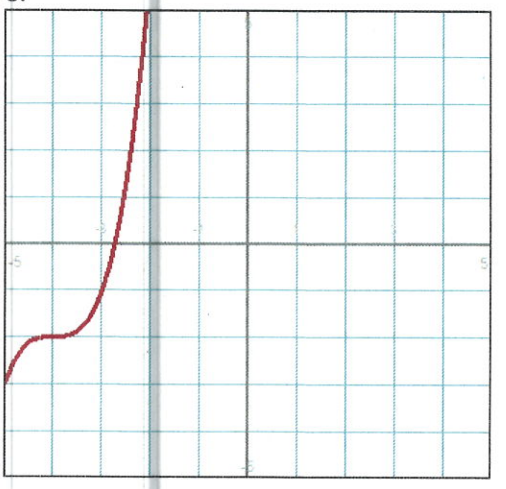

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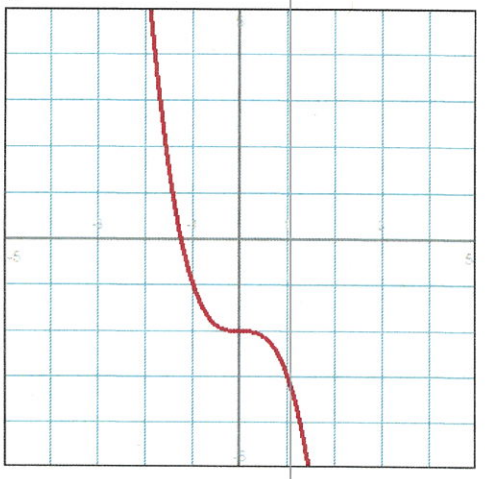
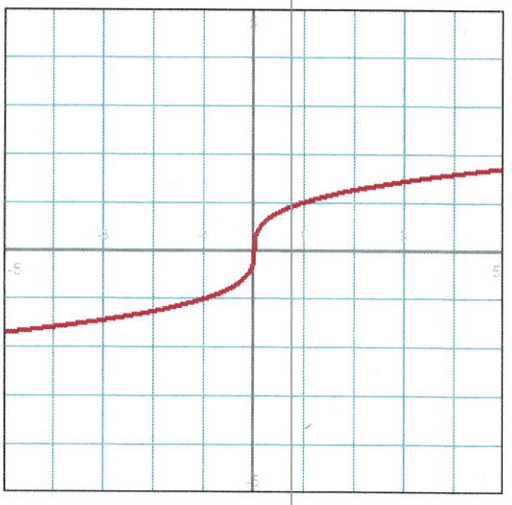


Base graph $l(x) = \sqrt[3]{x}$



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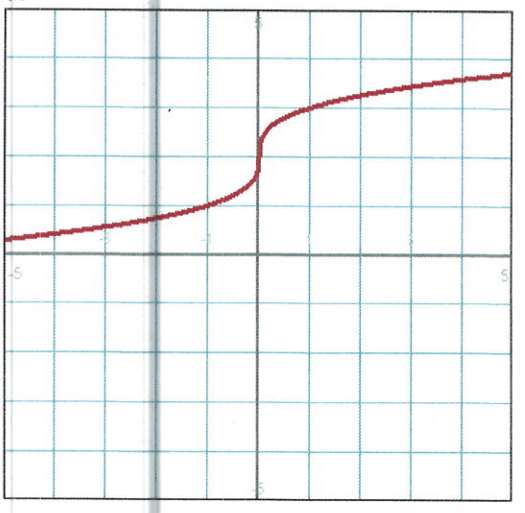

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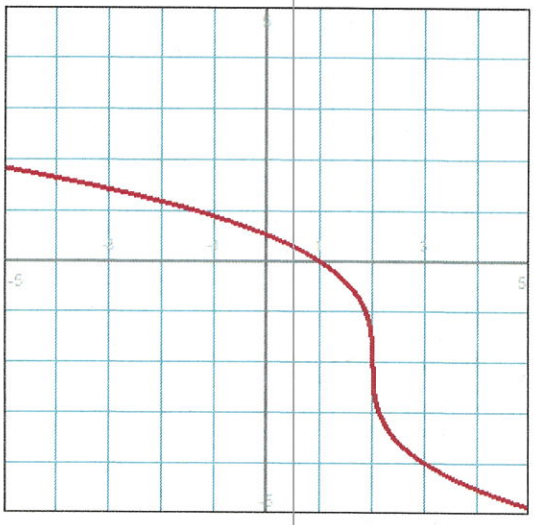

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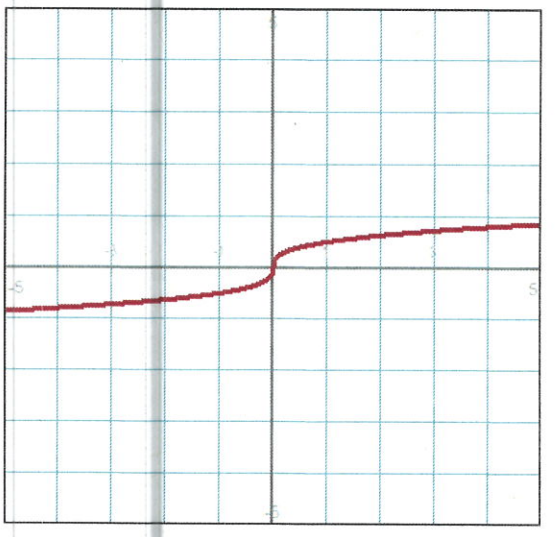

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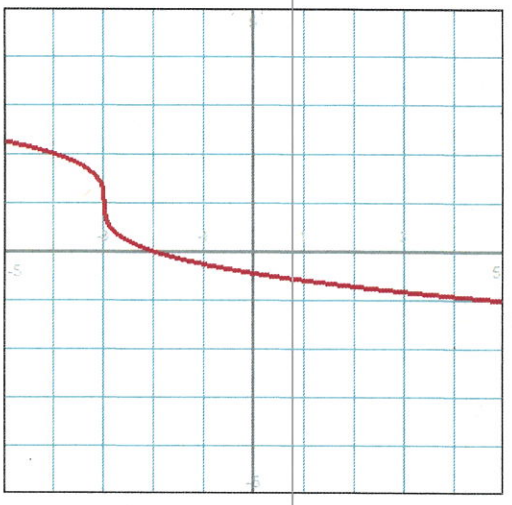

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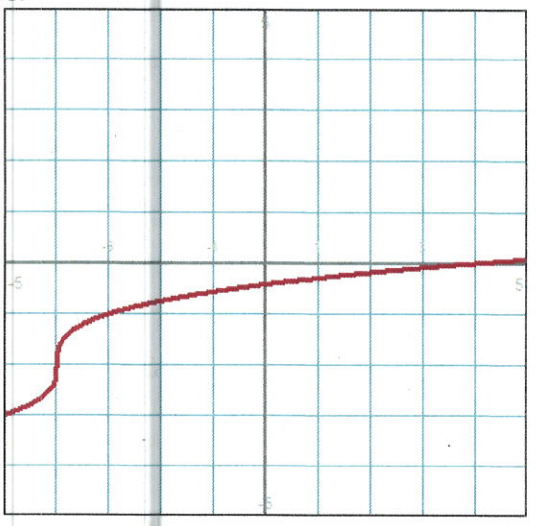

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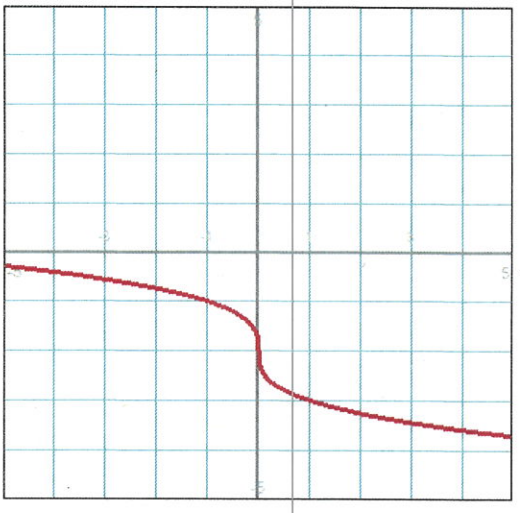
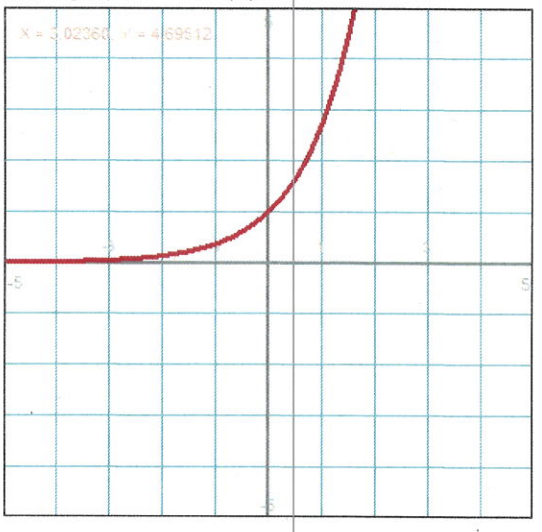


Base graph $m(x) = e^x$



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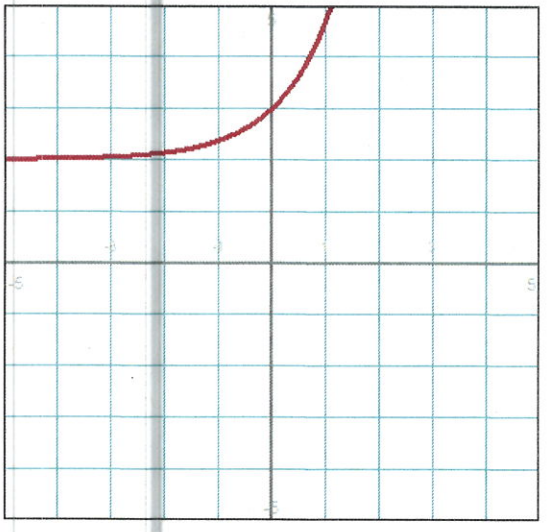

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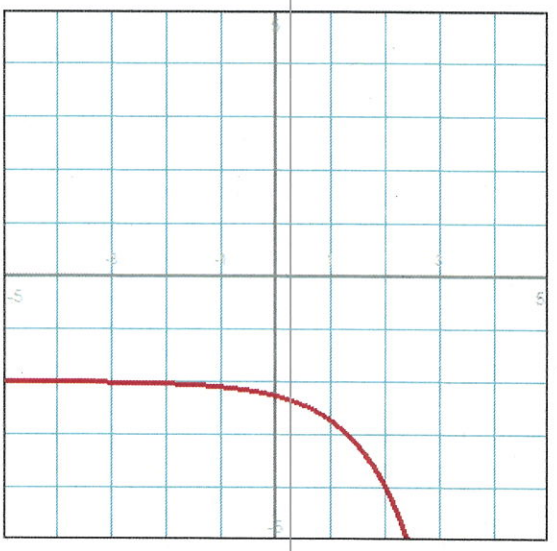

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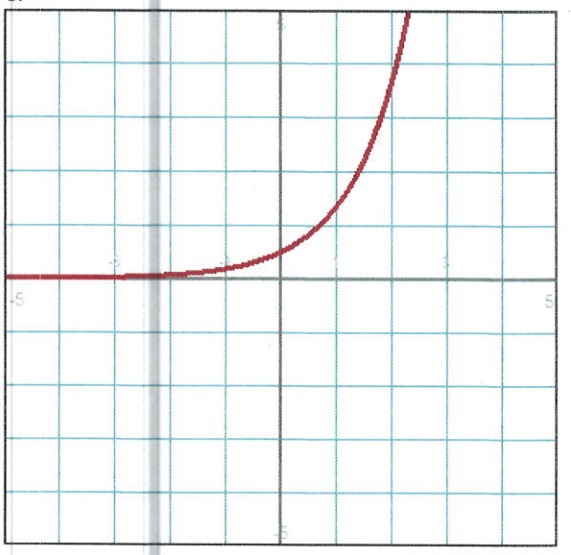

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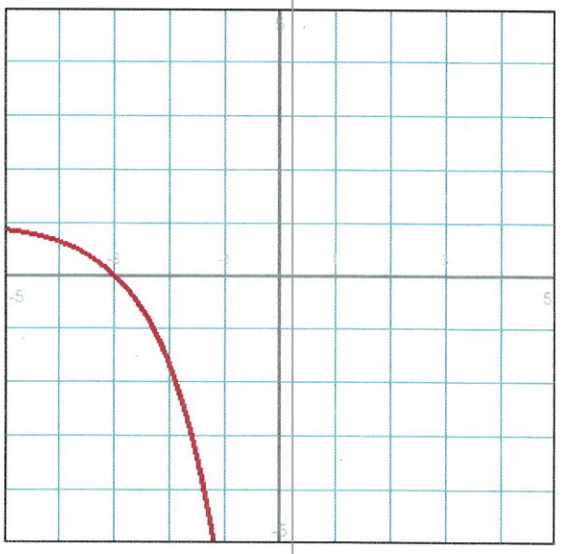

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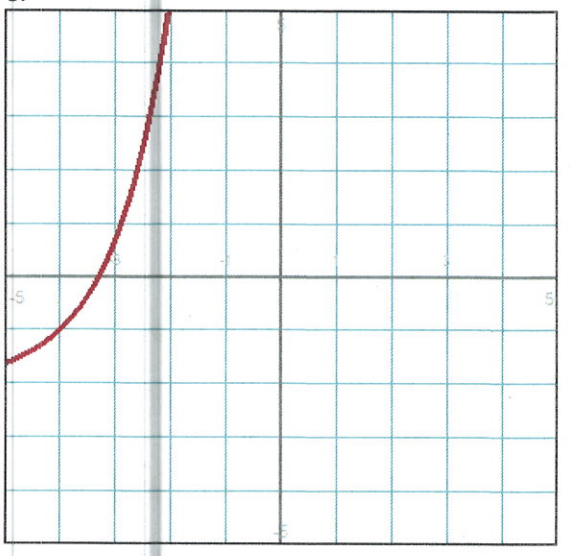

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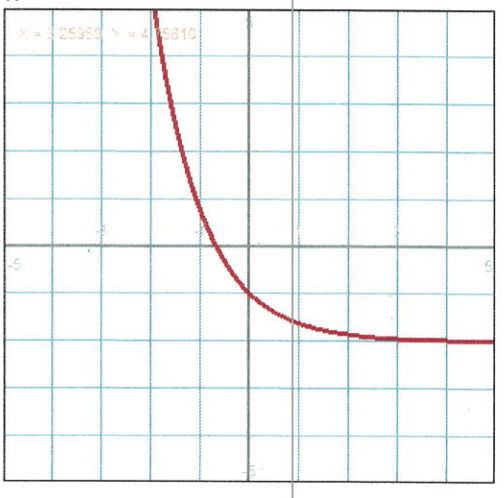
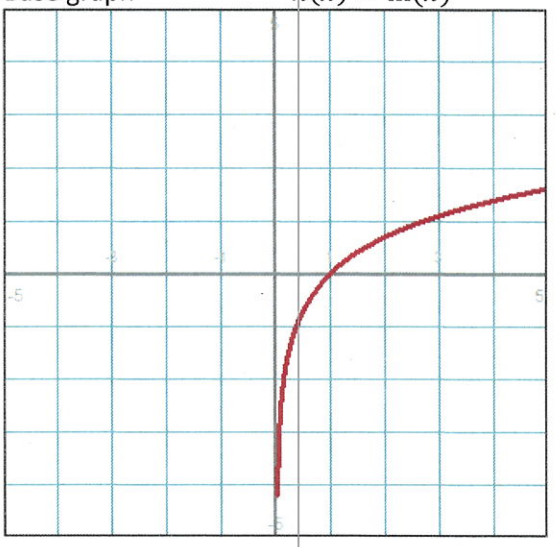
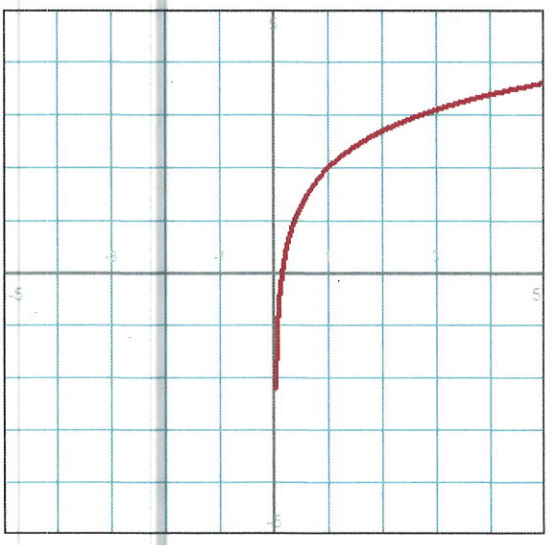


Base graph $n(x) = \ln(x)$



a.



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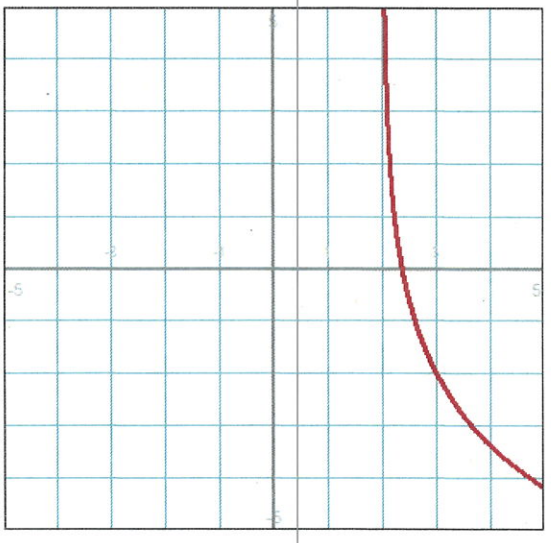

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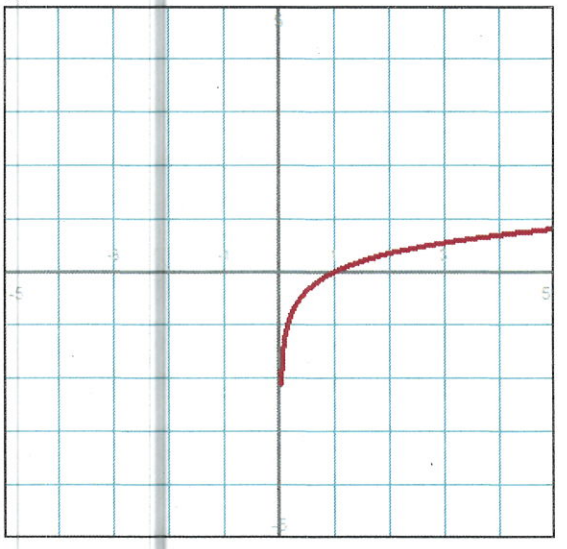

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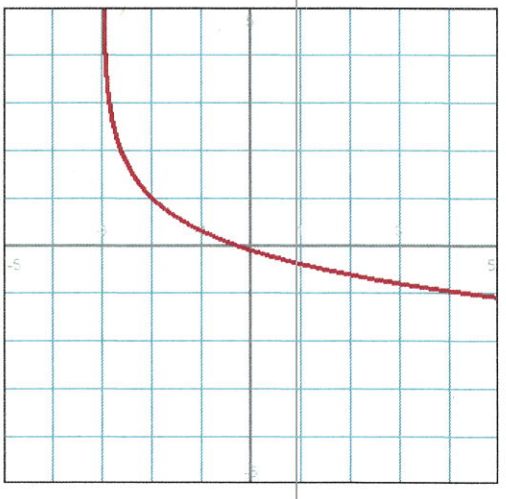

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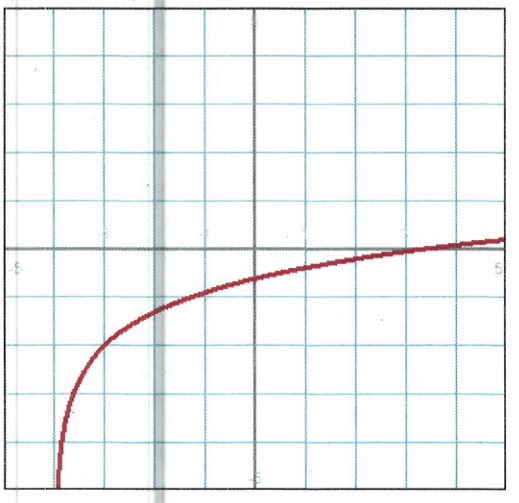

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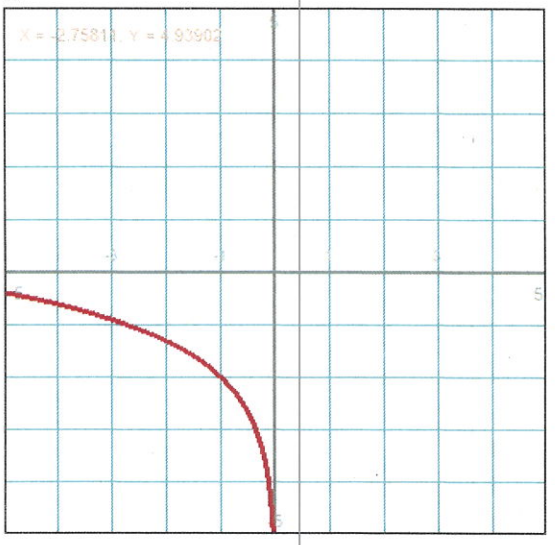


In words:

a. Vertical shift up 2
b. Horizontal shift right 2, vertical reflection, vertical stretch by factor of 2, vertical shift down by 2
c. Vertical compression by ½
d. Horizontal shift left 3, vertical reflection, vertical shift up 1
e. Horizontal shift left 4, vertical shift down 2
f. Horizontal reflection, vertical shift down 2

3 cont'd

d. $g \circ f = (2x^2 - x + 3) + 1 = 2x^2 - x + 4$

D: $(-\infty, \infty)$

R: $[3.875, \infty)$

e. $g \circ g = (x+1) + 1 = x + 2$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

f. $f - g = 2x^2 - x + 3 - (x+1) = 2x^2 - 2x + 2$

D: $(-\infty, \infty)$

R: $[1.5, \infty)$

g. $\frac{f}{g} = \frac{2x^2 - x + 3}{x + 1} = 2x - 3$ D: $x \neq -1$, R: $y \neq -5$

h. $f \circ g = 2(x+1)^2 - (x+1) + 3 = 2(x^2 + 2x + 1) - x - 1 + 3 =$

$2x^2 + 4x + 2 - x - 1 + 3 = 2x^2 + 3x + 4$ D: $(-\infty, \infty)$ R: $[2.875, \infty)$

i. $f \circ f = 2(2x^2 - x + 3)^2 - (2x^2 - x + 3) + 3$

$2(4x^4 + 4x^3 + 12x^2 + x^2 - 6x + 9) - 2x^2 + x - 3 + 3$

$8x^4 + 8x^3 + 26x^2 - 12x + 18 - 2x^2 + x - 3 + 3$

$8x^4 + 8x^3 + 24x^2 - 11x + 18$ D: $(-\infty, \infty)$ R: $[16.836525, \infty)$

4. a. $\frac{5x+1}{x^2 - 9} + \sqrt{x - 2} = f + g$ D: $x \neq 3$, [illegible] and $x \geq 2$ R: $(-\infty, -2.2) \cup (3.1285, \infty)$

b. $fg = \frac{(5x+1)\sqrt{x-2}}{x^2 - 9}$ D: $x \geq 2$, $x \neq 3$ R: $(-\infty, \infty)$

c. $\frac{g}{f} = \frac{(\sqrt{x-2})(x^2 - 9)}{5x + 1}$ D: $x \geq 2$ R: $[0, \infty)$

d. $g \circ f = \sqrt{\frac{5x+1}{x^2 - 9} - 2}$ D: $(-3, -2) \cup (3, 9/2]$

R: $[0, \infty)$

e. $g \circ g = \sqrt{\sqrt{x-2} - 2}$ D: $[6, \infty)$ R: $[0, \infty]$

f. $f - g = \frac{5x+1}{x^2 - 9} - \sqrt{x - 2}$ D: $x \neq 3$, $x \geq 2$ R: $(-\infty, \infty)$

g. $\frac{f}{g} = \frac{5x+1}{(x^2 - 9)\sqrt{x-2}}$ D: $[2, 3) \cup (3, \infty)$ R: $(-\infty, -6.1511..) \cup (0, \infty)$

4.h. $f \circ g = \dfrac{5\sqrt{x-2}+1}{(x-2)-9} = \dfrac{5\sqrt{x-2}+1}{x-11}$

D: $x \geq 2,\ x \neq 11$

R: $(-\infty, 0]$

i. $f \circ g = \dfrac{5\left(\frac{5x+1}{x^2-9}\right)+1}{\left(\frac{5x+1}{x^2-9}\right)^2-9}$

R: $(-\infty, \infty)$

D: $x \neq 3, -3, 4, 2.2\overline{2}, -2.3\overline{3}, -3.89\ldots$

5. answers may vary

a. $h(x) = (3x-1)^4$ $\quad f(x) = x^4,\ g(x) = 3x-1$

$f \circ g = h(x)$

b. $h(x) = |2x-5|$ $\quad f(x) = |x|,\ g(x) = 2x-5$

$f \circ g = h(x)$

c. $h(x) = \sqrt{5x^2+3}$ $\quad f(x) = \sqrt{x},\ g(x) = 5x^2+3$

$f \circ g = h(x)$

d. $h(x) = \dfrac{1}{4x+5}$ $\quad f(x) = \dfrac{1}{x},\ g(x) = 4x+5$

$f \circ g = h(x)$

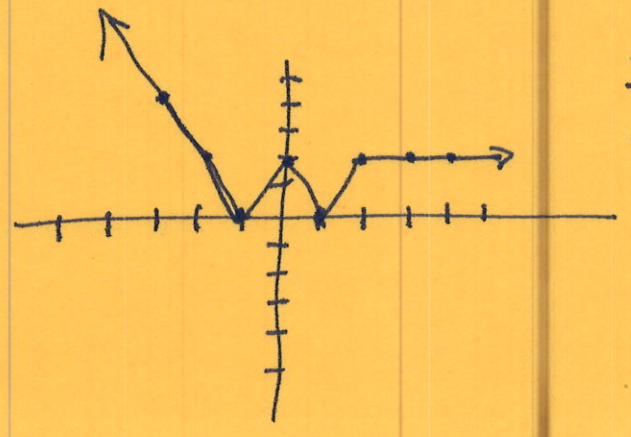
6. a. $(f+g)(-3) = f(-3) + g(-3) = 3+3 = 6$

b. $\left(\dfrac{g}{f}\right)(3) = \dfrac{g(3)}{f(3)} = \dfrac{2}{3} = \dfrac{2}{3}$

d. $(fg)(2) = f(2) \cdot g(2) = 2 \cdot 2 = 4$

e. $(f \circ g)(1) = f(g(1)) = f(1) = 1$

c. graph of $f \circ g(x)$



x	g(x)	f(g(x))
-2	2	2
-1	0	0
0	-2	2
1	0	0
2	2	2
3	2	2
4	2	2
-3	4	4