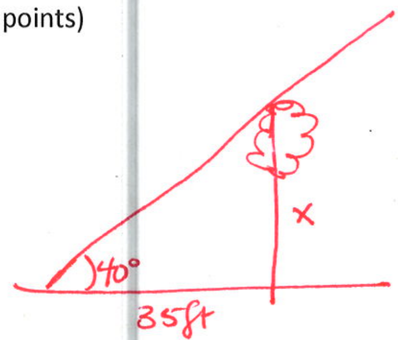
MAT 142, Exam #2, Spring 2017 Name KEY

Instructions: Show all work. Give exact answers unless specifically asked to round. Complete all parts of each question. Questions that provide only answers and no work will not receive full credit. If you use your calculator (only when problems don't instruct you to do the problem by hand), showing calculator steps will count as "work".

1. At a certain time of day, the angle of elevation of the sun is 40°. To the nearest foot, find the height of a tree whose shadow is 35 feet long. (4 points)



$\frac{x}{35} = \tan 40°$

$x = 35 \tan 40° = 29.368 \Rightarrow \boxed{29 ft}$

2. Find the exact value of the six trig functions if the coterminal side of the angle passes through the point $(-1, -3)$. (4 points)

x y $r = \sqrt{10}$

$\sin\theta = \frac{-3}{\sqrt{10}}$ $\sec\theta = -\sqrt{10}$

$\cos\theta = \frac{-1}{\sqrt{10}}$ $\csc\theta = \frac{-\sqrt{10}}{3}$

$\tan\theta = 3$

$\cot\theta = 1/3$

3. Use a reference angle to find the exact value of each of the following. (3 points each)
   a. $\sin\left(-\frac{35\pi}{6}\right)$

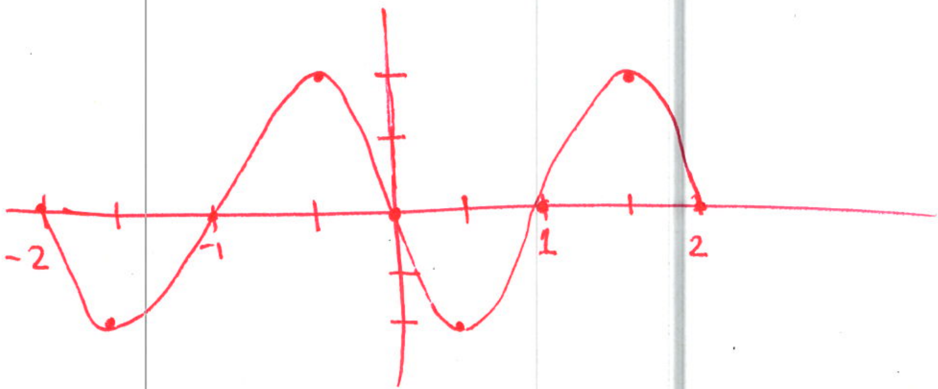
      $1/2$

   b. $\tan 210°$ $\frac{1}{\sqrt{3}}$

4. Use key points to graph two periods of each function, by hand, using key points. (6 points each)

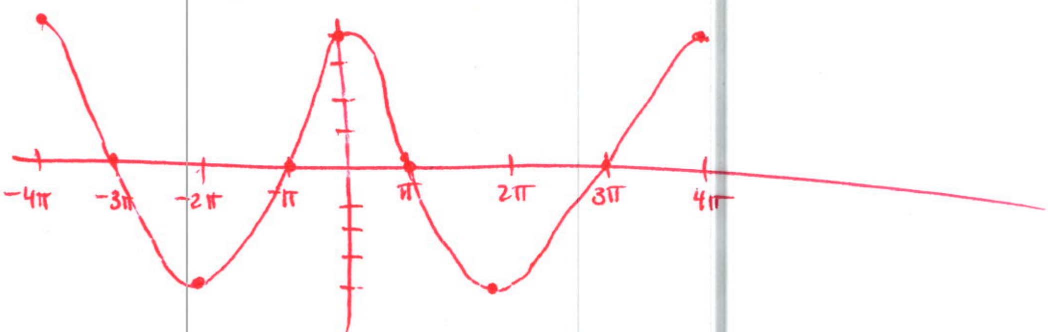
a. $y = -2\sin \pi x$

$\text{period} = \frac{2\pi}{\pi} = 2$ $\qquad$ amplitude $= 2$



b. $y = 4\cos\frac{1}{2}x$

$\text{period} = \frac{2\pi}{1/2} = 4\pi$



5. For each function, state the amplitude, period, phase shift and any vertical shift. (3 points each)

a. $y = \frac{-1}{2}\cos(2x + \pi)$

$[2(x + \pi/2)]$

amp $= \frac{1}{2}$ $\quad$ period $= \frac{2\pi}{2} = \pi$

phase shift $= -\pi/2$

no vertical shift

b. $y = -3\sin 2\pi x + 2$

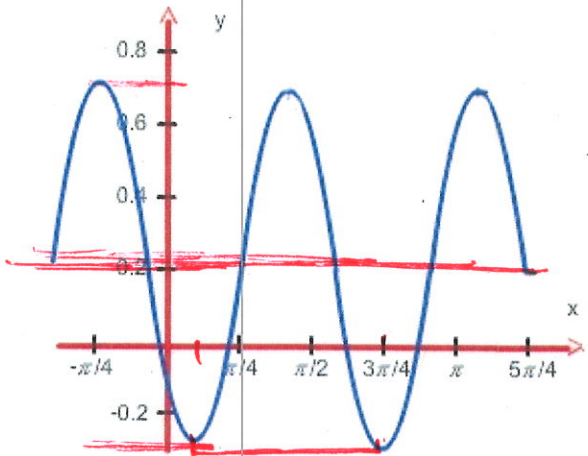
vertical shift up 2

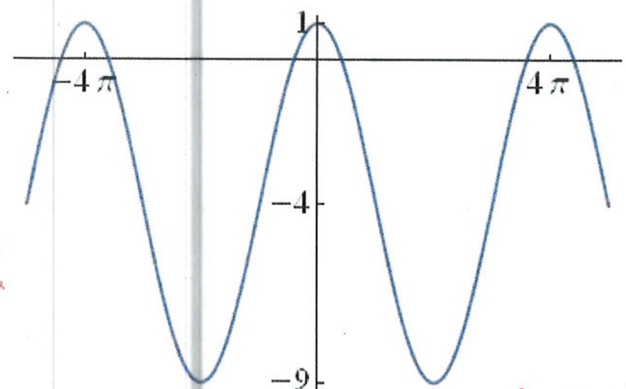
no phase shift

amplitude $= 3$

period $= \frac{2\pi}{2\pi} = 1$

6. Write an equation of the graph. (4 points each)





$\frac{5\pi}{8}$

answers may vary

amplitude $\frac{.7-(-.30)}{2} = \frac{1}{2}$

period $= \frac{5\pi}{8}$ $\quad \omega = 2\pi \cdot \frac{8}{5\pi} = \frac{16}{5}$

phase shift $-\pi/4$

$\boxed{y = \frac{1}{2}\cos\left[\frac{16}{5}\left(x+\pi/4\right)\right] + 1/5}$

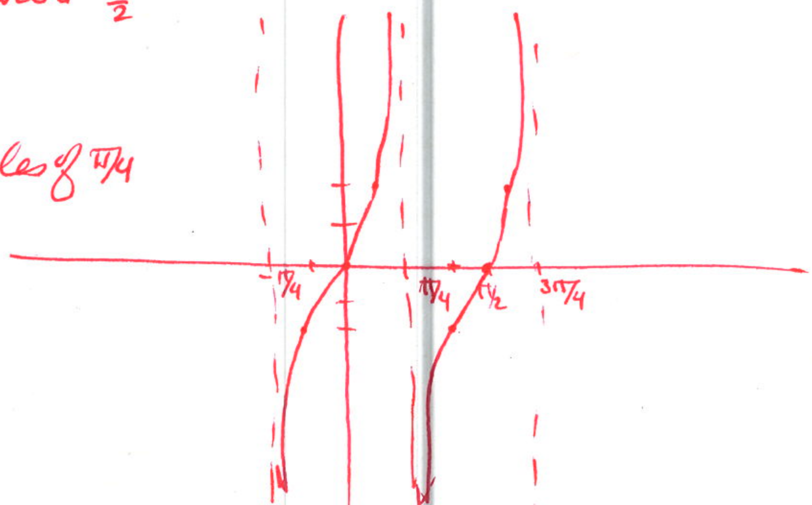
ampl. $= 10/2 = 5$
no phase shift
period $= 4\pi$
vertical shift -4

$\boxed{y = 5\cos\frac{1}{2}x - 4}$

7. Graph the functions for 2 periods, by hand, using key points. State the domain of each. (6 points each)

a. $y = 2\tan 2x$ period $\frac{\pi}{2}$

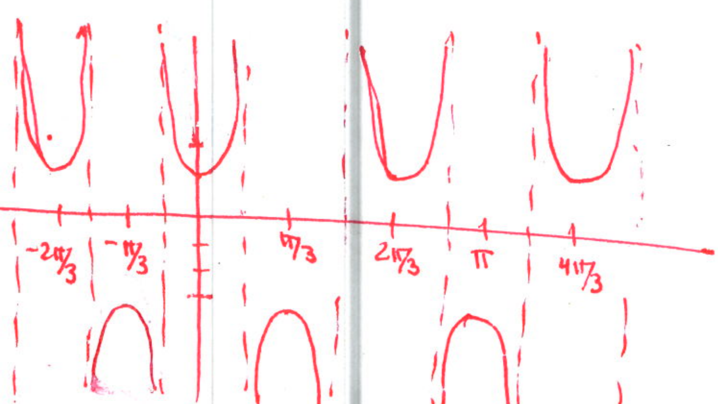
domain
$x \neq$ odd multiples of $\pi/4$



b. $y = 2\sec 3x - 1$

period $= \frac{2\pi}{3}$

domain:
$x \neq$ odd multiples of $\pi/6$



8. Find the exact value of each expression. (3 points each)

a. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

$5\pi/6$

b. $\tan^{-1}(1)$

$\pi/4$

c. $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$-\pi/4$

d. $\tan\left(\sin^{-1}\left(-\frac{3}{5}\right)\right)$

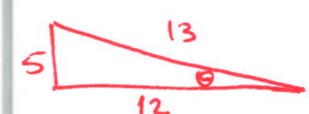
$-\frac{3}{4}$

e. $\csc\left(\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right)$

$= 2$

f. $\cot\left(\sin^{-1}\frac{5}{13}\right)$

$\frac{12}{5}$



9. Find the domain and range of $y = \sin^{-1}(x-2) + \frac{\pi}{2}$. (5 points)

Sine^{-1} domain $[-1, 1]$ shift right by 2 $\rightarrow$ $\boxed{[1, 3] \text{ domain}}$

sine^{-1} range $[-\pi/2, \pi/2]$ shift up by $\pi/2$ $\rightarrow$ $\boxed{[0, \pi] \text{ range}}$

10. Two triangles can be formed with $a = 30, b = 40, A = 20°$. Find all the missing sides and angles of both. (7 points)

$$\frac{\sin 20°}{30} = \frac{\sin B}{40}$$

$$\sin B = \frac{40 \sin 20°}{30} = .456...$$

$$B = 27.13° \text{ or } 152.87°$$

$20° + 27.13° = 47.13$

$180 - 47.13 = 132.87° = C$

$$\frac{\sin 20°}{30} = \frac{\sin 132.87°}{c}$$

$$c = \frac{30 \sin 132.87°}{\sin 20°} = 64.3$$

$20° + 152.87° = 172.87°$

$180 - 172.87 = 7.13° = C$

$$\frac{\sin 20°}{30} = \frac{\sin 7.13}{c} \Rightarrow c = \frac{30 \sin 7.13°}{\sin 20°} = 10.887 = 10.9$$

triangle 1: $a = 30, b = 40, c = 64.3, A = 20°, B = 27.13°, C = 132.87°$

triangle 2: $a = 30, b = 40, c = 10.9, A = 20, B = 152.87, C = 7.13$

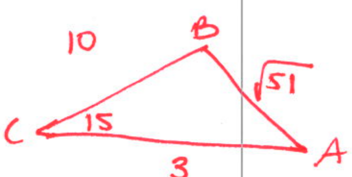
11. A leaning wall is inclined 6° from the vertical. At a distance of 40 feet from the wall, the angle of elevation to the top is 22°. Find the height of the wall to the nearest tenth of a foot. (Assume that the lean of the wall is in the direction of where the angle of elevation was measured.) (6 points)

$$\frac{\sin 22°}{X} = \frac{\sin 74°}{40}$$

$$\frac{40 \sin 22°}{\sin 74°} = X$$

$X = 15.6$ ft.

12. Find the missing sides and angles of a triangle with $a = 10, b = 3, C = 15°$. (6 points)



$a^2 + b^2 - 2ab \cos C = c^2$

$3^2 + 10^2 - 2(3)(10) \cos 15 = 51.$

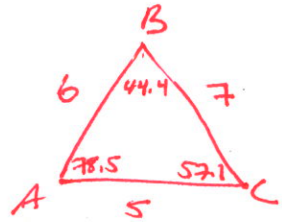
$c = \sqrt{51}$

$$\frac{\sin 15}{\sqrt{51}} = \frac{\sin A}{10}$$

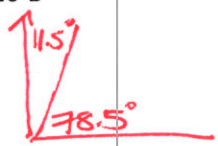
$$\frac{10 \sin 15}{\sqrt{51}} = \sin A$$

$A = 21.2°$ $B = 143.8°$

13. Three islands form a triangle. The distance between islands A and B is 6 miles; the distance between islands B and C is 7 miles, and the distance between islands A and C is 5 miles. If you are on the given island, find the bearing to the second island. (10 points)



a. A to B



N11.5°E

b. B to C

S32.9°E

$a^2+b^2-2ab\cos C=c^2$

$\frac{c^2-a^2-b^2}{-2ab}=\cos C$

$C=57.1°$

c. C to A

W

$\frac{\sin 57.1}{6}=\frac{\sin A}{7}$

$\frac{7\sin 57.1}{6}=\sin A$

$A=78.5°$

$B=44.4°$

answers may vary in bearing depending on orientation of triangle

14. Verify the identities. (4 points each)

a. $\frac{\sin x}{1+\cos x}+\frac{\cos x-1}{\sin x}=0$

$\frac{(\cos x-1)\sin x}{(\cos x-1)(1+\cos x)}+\frac{(\cos x-1)\sin x}{\sin^2 x}$

$\cos^2 x-1=-\sin^2 x$

$\frac{(1-\cos x)\sin x}{\sin^2 x}+\frac{(\cos x-1)\sin x}{\sin x}=0$ ✓

b. $(\sec x-\tan x)^2=\frac{1-\sin x}{1+\sin x}$

$\sec^2 x-2\sec x\tan x+\tan^2 x=$

$\frac{1}{\cos^2 x}-\frac{2\sin x}{\cos^2 x}+\frac{\sin^2 x}{\cos^2 x}=$

$\frac{1-2\sin x+\sin^2 x}{\cos^2 x}=\frac{(1-\sin x)^2}{1-\sin^2 x}=\frac{(1-\sin x)\cancel{(1-\sin x)}}{(1+\sin x)\cancel{(1-\sin x)}}=\frac{1-\sin x}{1+\sin x}$

c. $\sin^4 t - \cos^4 t = 1 - 2\cos^2 t$

$$(\sin^2 t - \cos^2 t)\underbrace{(\sin^2 t + \cos^2 t)}_{=1} = -1(\cos^2 t - \sin^2 t)$$

$$= -1(2\cos^2 t - 1) = 1 - 2\cos^2 t$$

d. $\frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 1 + \sin x \cos x$

$$\frac{\cancel{(\sin x - \cos x)}(\sin^2 x + \sin x \cos x + \cos^2 x)}{\cancel{\sin x - \cos x}} = \sin^2 x + \cos^2 x + \sin x \cos x$$

$$= 1 + \sin x \cos x$$

e. $\sin\left(x + \frac{3\pi}{2}\right) = -\cos x$

$$\sin x \cancel{\cos \frac{3\pi}{2}}^{=0} + \underbrace{\sin \frac{3\pi}{2}}_{=-1} \cos x = -\cos x$$

f. $\sin^2 \frac{\theta}{2} = \frac{\sec\theta - 1}{2\sec\theta}$

$$\sin^2 \frac{\theta}{2} = \frac{1-\cos\theta}{2}\left(\frac{\sec\theta}{\sec\theta}\right) = \frac{\sec\theta - 1}{2\sec\theta}$$

g. $\sin 2t - \tan t = \tan t \cos 2t$

$$\left(\frac{\cos t}{\cos t}\right) 2\sin t \cos t - \frac{\sin t}{\cos t} = \frac{2\sin t \cos^2 t - \sin t}{\cos t} = \frac{\sin t}{\cos t}(2\cos^2 t - 1)$$

$$= \tan t \cos 2t$$

15. Use identities to find exact values for each of the following. (3 points each)

a. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12}+\sin\frac{5\pi}{12}\sin\frac{\pi}{12}$

$$=\cos\left(\frac{5\pi}{12}-\frac{\pi}{12}\right)=\cos\left(\frac{4\pi}{12}\right)=\cos\left(\frac{\pi}{3}\right)=\boxed{\frac{1}{2}}$$

b. $\sin 75° = \sin(30°+45°)$

$$=\sin 30°\cos 45° + \sin 45°\cos 30°$$

$$\frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

c. $\cos 22.5°$ $\quad \frac{45°}{2}=22.5$

$$\cos\frac{45°}{2}=\sqrt{\frac{1+\cos 45°}{2}}=\sqrt{\frac{1+\sqrt{2}/2}{2}}=\sqrt{\frac{2}{4}+\frac{\sqrt{2}}{4}}=\boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$$

d. $\tan\frac{7\pi}{8}$ $\quad \frac{7\pi}{8}=\left(7\pi/4\right)/2$

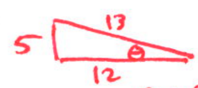
$$\tan\frac{7\pi}{8}=\frac{1-\cos 7\pi/4}{\sin 7\pi/4}=\frac{1-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}}\left(\frac{2}{2}\right)=\frac{2-\sqrt{2}}{-\sqrt{2}}=\frac{\sqrt{2}-2}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=$$

$$\frac{2-2\sqrt{2}}{2}=\boxed{1-\sqrt{2}}$$

16. For $\cos\theta=\frac{12}{13}$, and θ in Q IV, find each of the following. (3 points each)

a. $\sin 2\theta$

$$2\left(-5/13\right)\left(\frac{12}{13}\right)=\boxed{\frac{-120}{169}}$$



$\sin\theta = -5/13$

b. $\tan\frac{\theta}{2}$

$$\frac{1-\frac{12}{13}}{\frac{-5}{13}}\left(\frac{13}{13}\right)=\frac{13-12}{-5}=\boxed{\frac{-1}{5}}$$

17. Solve for all values of the variable in $[0,2\pi)$. (5 points each)

a. $\sin 2x = \sin x$

$2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \Rightarrow x = 0, \pi$

$2\cos x - 1 = 0 \Rightarrow$

$\cos x = \frac{1}{2}$

$x = \pi/3, 5\pi/3$

$\boxed{\{0, \pi/3, \pi, 5\pi/3\}}$

b. $\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\sin\alpha = \frac{\sqrt{2}}{2}$

$\alpha = \pi/4, 3\pi/4$

$2x - \frac{\pi}{4} = \pi/4$

$2x = \pi/2$

$x = \pi/4, 5\pi/4$

$2x - \pi/4 = 3\pi/4$

$2x = \pi$

$x = \pi/2, 3\pi/2$

$\boxed{\{\pi/4, 5\pi/4, \pi/2, 3\pi/2\}}$

c. $3\cos^2 x = \sin^2 x$

$3\cos^2 x = 1 - \cos^2 x$

$4\cos^2 x = 1$

$\cos^2 x = 1/4$

$\cos x = \pm\frac{1}{2}$

$\boxed{x = \pi/3, 2\pi/3, 4\pi/3, 5\pi/3}$

Some useful formulas:

$$\sin(a+b) = \sin a \cos b + \sin b \cos a$$
$$\sin(a-b) = \sin a \cos b - \sin b \cos a$$
$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

$$\begin{aligned} \sin 2\alpha &= 2\sin\alpha\cos\alpha \\ \cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= 2\cos^2\alpha - 1 \\ &= 1 - 2\sin^2\alpha \\ \tan 2\alpha &= \frac{2\tan\alpha}{1-\tan^2\alpha} \end{aligned}$$

$$\sin(\frac{a}{2}) = \pm\sqrt{\frac{(1-\cos a)}{2}}$$
$$\cos(\frac{a}{2}) = \pm\sqrt{\frac{(1+\cos a)}{2}}$$
$$\tan(\frac{a}{2}) = \frac{1-\cos a}{\sin a} = \frac{\sin a}{1+\cos a}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$
$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$
$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$