

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each problem.

1. Convert the following preference ballots into a preference schedule (table). (8 points)

Ballot 1st C 2nd A 3rd D 4th B ①	Ballot 1st B 2nd C 3rd D 4th A ②	Ballot 1st A 2nd D 3rd B 4th C ③	Ballot 1st C 2nd A 3rd D 4th B ①	Ballot 1st B 2nd C 3rd D 4th A ②
Ballot 1st A 2nd D 3rd B 4th C ③	Ballot 1st A 2nd C 3rd D 4th B ④	Ballot 1st B 2nd C 3rd D 4th A ②	Ballot 1st B 2nd C 3rd D 4th A ②	Ballot 1st C 2nd A 3rd D 4th B ①
Ballot 1st A 2nd D 3rd D 4th B ④	Ballot 1st A 2nd D 3rd B 4th C ③	Ballot 1st C 2nd A 3rd D 4th B ①	Ballot 1st B 2nd C 3rd D 4th A ②	Ballot 1st A 2nd D 3rd B 4th C ③
				Ballot 1st C 2nd A 3rd D 4th B ①

Voters	6	5	4	2
1st	C	B	A	A
2nd	A	C	D	C
3rd	D	D	B	D
4th	B	A	C	B

2. Use the following preference schedule to find the winner of the election using the indicated method.

Number of voters	14	10	8	7	4
Andersson	2	3	1	5	3
Broderick	1	1	2	3	2
Clapton	4	5	5	2	4
Dutkiewicz	5	2	4	1	5
Eklundh	3	4	3	4	1

a. Plurality Method (6 points)

A: 8 C: 0 E: 4 B wins
 B: 24 D: 7

b. Borda Count Method (10 points)

$A: 14 \times 4 + 10 \times 3 + 8 \times 5 + 7 \times 1 + 4 \times 3 = 145$
 $B: 14 \times 5 + 10 \times 5 + 8 \times 4 + 7 \times 3 + 4 \times 3 = 185 \leftarrow \text{winner}$
 $C: 14 \times 2 + 10 \times 1 + 8 \times 1 + 7 \times 4 + 4 \times 1 = 78$
 $D: 14 \times 1 + 10 \times 4 + 8 \times 2 + 7 \times 5 + 4 \times 1 = 79$
 $E: 14 \times 3 + 10 \times 2 + 8 \times 3 + 7 \times 2 + 4 \times 5 = 120$

c. Plurality with Elimination Method (8 points)

Rnd 1
 A: 8
 B: 24
 C: 0
 E: 4
 D: 7

B wins
 no need for additional
 rounds since B already
 has a majority
 $48/2 = 24 \rightarrow 22$ majority
 24 > 22

d. Method of Pairwise Comparisons (9 points)

$A \vee B$ 14+10
 $A \vee C$ 14+10
 $A \vee D$ 14+8 10
 $A \vee E$ 14+10
 $B \vee C$ 24
 $B \vee D$ 24
 $B \vee E$ 24
 $C \vee D$ 14 10+8+7
 $C \vee E$ 14+10
 $D \vee E$ 10 14+8

A B C D E
 III III 1 II

B wins

e. Is there a majority criterion violation? Why or why not? (5 points)

no. B has a majority, but B won all elections

f. Is there a Condorcet criterion violation? Why or why not? (5 points)

no, since B wins pairwise comparison and all other methods

3. What is the difference between a majority and a plurality? (4 points)

a majority is 50% + 1 vote

a plurality is the most votes obtained.

they are only the same when there are 2 candidates

4. Explain Arrow's Impossibility Theorem. How is it similar to Young's Impossibility Theorem? (7 points)

This theorem says that no voting system is always completely fair. All methods will violate some fairness criterion at least some of the time

5. A County Elections Board needs to allocate 50 "floating" pollworkers to various communities during an election to troubleshoot problems that arise. The communities and their registered voting populations are noted in the table below. Use the tables to apportion the pollworkers to the various communities.

Town	Population	Standard Quota	Lower Quota	Upper Quota	Extra Seat?	Final Apportionment
Oakcastle	3182	9.084	9	10		9
Southaven	2509	7.163	7	8		7
Whitefaire	4011	11.451	11	12		11
Easthill	3703	10.572	10	11	+1	11
Eriden	4109	11.731	11	12	+1	12
Standard Divisor =		350.28	48			50

17514
total
pop

a. By Hamilton's Method (10 points)

Town	Population	Standard Quota	Modified Quota	Modified Quota	Modified Quota	Final Apportionment
Oakcastle	3182	9.084	9.4985			9
Southhaven	2509	7.163	7.4895			7
Whitefaire	4011	11.451	11.973			11
Easthill	3703	10.572	11.053			11
Eriden	4109	11.731	12.26			12
Standard Divisor = 350.28						50

b. By Jefferson's Method (10 points)

You may use one of the modified quotas here (one of them will work): 352.5, 370, 335.

Town	Population	Standard Quota	Lower Quota	Upper Quota	Geometric Mean	Final Apportionment
Oakcastle	3182	9.084	9	10	9.486	9
Southhaven	2509	7.163	7	8	7.483	7
Whitefaire	4011	11.451	11	12	11.489	11
Easthill	3703	10.572	10	11	10.488	11
Eriden	4109	11.731	11	12	11.489	12
Standard Divisor = 350.28						50

c. By Huntington-Hill's Method. (10 points)

d. Do any of these apportionment methods produce a quota rule violation in this instance? (4 points)

none of the produce quota rule violations in this instance

6. Which apportionment paradox is illustrated in the table below? Or is it a paradox? If it is, state which one it is, and show calculations to justify your conclusion. If it is not, explain why not (and show calculations to justify your conclusion)..(7 points each)

	Population		Seats	
	1900	1901	1900	1901
Maine	694466	699114	3	4
Virginia	1854184	1873951	10	9

.00669 4698

.01066 19767a.

Virginia grew at a higher rate but
lost a seat
population paradox

	population	initial apportionment (after rounding down)	Modified quotas	Modified lower quotas	Final apportionment
A	1646	$1646/50 = 32$ ≈ 33	33.25	33	33
B	6936	$6936/50 = 138$ ≈ 139	140.12	140	140
C	154	$154/50 = 3$ ≈ 4	3.11	3	3
D	2091	$2091/50 = 41$ ≈ 42	42.24	42	42
E	685	$685/50 = 13$ ≈ 14	13.84	13	13
F	988	$988/50 = 19$ ≈ 20	19.95	19	19
total	12,500	246 (so 4 seats remain)	252.5	250	250

b.

quota rule violation

140 is bigger than the upper quota
of 139 for B