Modeling Springs
Math 212, Spring 2016

When modeling springs mathematically, textbooks typically begin with ideal “massless” springs. For this activity we will be working with real springs, and all the potential complications that come with real world models. For this activity, divide up into groups of 2-5 students. The number of groups may depend on the number of springs available and the number of motion detectors. Throughout this activity, you will periodically be asked to check with your instructor. Please do not skip these steps!


A. Select a spring to being working with. Describe below the features of the spring: size, material, etc.

B. Hang the spring from the ceiling, or other stable shelf where it can hang freely and not knock into people, walls or furniture. When the spring settles in to equilibrium, sketch the spring below. What do you notice about the spacing of the coils of the spring in its resting state?

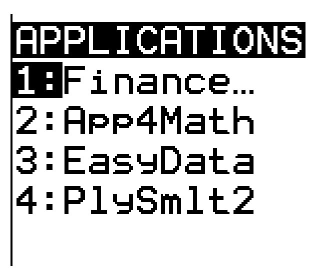
C. Add a small amount of mass to the cup hanging under the bottom of the spring. Record the amount added below. When the spring achieves equilibrium, what do you notice about the spring now? How has it changed from above?

Check with your instructor.

D. Pull the spring/mass down from equilibrium and release the spring. Watch the position of the mass change. Sketch a diagram below of the motion of the spring/mass system through at least two full oscillations. Be sure to label your axes. What do you notice over time?

E. Consult with your instructor about setting up and using the Go! Motion (motion detector). To download your data to your TI calculator, go to 

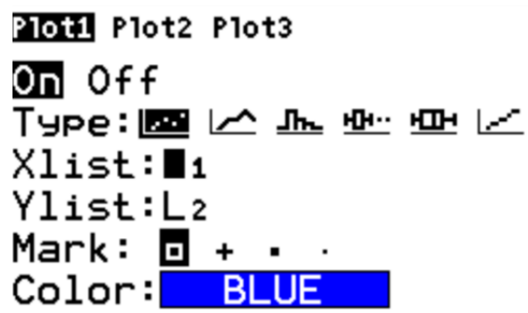
and click on **Easy Data** (it should be pre-installed in your calculator). Or, you can connect the Go! Motion to your tablet or computer to capture the data with Logger Lite (available for free from Vernier).[1]



[11] You can download Easy Data for your calculator here: http://www.vernier.com/platforms/texas-instruments/software/data-collection/easydata/ (you will first need to download TI Connect here: https://education.ti.com/en/us/products/computer_software/connectivity-software/ti-connect-software/tabs/overview#!lightbox=TI-Connect-TI-Connect-CE). Both programs are free. You can download Logger Lite here: http://www.vernier.com/products/software/logger-lite/

Set up the motion detector in TimeGraph mode, to take data every 0.05 seconds or more often. You will record for approximately 5 seconds, so this will ensure at least 100 data points. When you are ready, set the spring-mass system in motion and then start the motion detector to collect data. Check the data in your calculator (or on the computer screen). You should get a smooth curve. If you don't get a smooth curve, be sure you or other obstacles are not being tracked by the detector accidentally and that the motion of the spring is not chaotic; then try again.

Sketch the graph you obtain. (Easy Data will store the data in your calculator. Select STAT, and Edit… to see which list it is in. To draw the graph in the TI, select 2nd Y= to enter **StatPlot**. Turn on Plot1, press ENTER, and move the cursor to ON and press ENTER again. On the Type line, the first graph type is the scatterplot. Put the cursor over it and press ENTER to select. Change the **XList** to the list where your time data is stored, and change the **YList** to where your position data is stored.



How does this graph compare to your prediction?

F. What kind of wave is being measured? A transverse wave or a compression wave?

G. What is the closest the mass gets to the motion detector?

H. How far is the equilibrium from the detector?

I. What is the largest distance the mass gets from the detector?

J. The amplitude of the wave is the maximum distance the mass gets from the equilibrium. What is the amplitude of the system?

K. Does the amplitude of the motion depend on how far you start the mass from equilibrium? (If you use the motion detector to test this, be sure to save your data from part E in another list or file first.) If so, in what way?

L. Does the amplitude of the spring-mass system change over time?

M. The period of the motion is the time that it takes for the mass to complete one full cycle: either from highest point to highest point, or lowest point to lowest point. What is the period of your system? Does the period change over time?

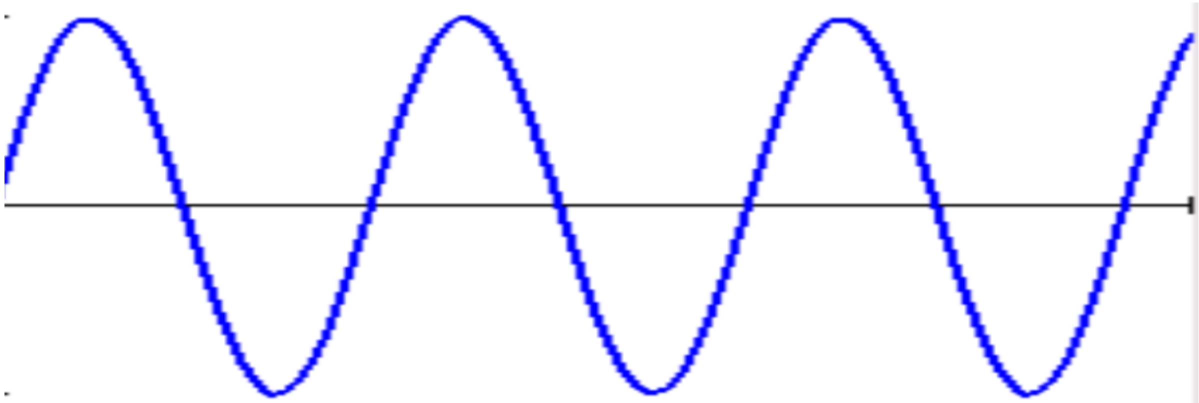
N. How many periods does your spring-mass system go through in the time you took data? How many times did your mass pass equilibrium?

O. The frequency of the system is the number of cycles completed in one second. What is the frequency of your system? [One cycle per second = Hertz. What is your frequency in Hertz?]

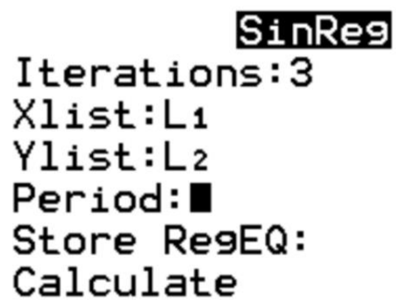
P. Use your answers in N and O to determine how the frequency of the system and the period of the system are related.

Check with your instructor.

Q. The graph of a sinusoidal “wave” is shown below. Label key points on this graph with your period, frequency, amplitude and equilibrium from your data.



R. Assuming the amplitude remains constant over time, we can write the equation of the position of the mass as $y(t) = A\sin(\omega t + \delta) + C$. Store this equation in Y1. Confirm your results by using the Sine Regression function in your calculator as follows:

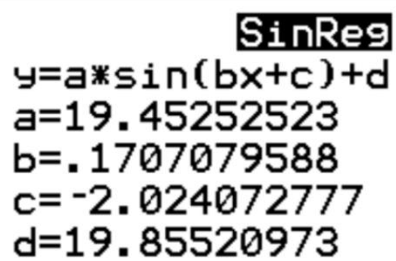


Press 

, then scroll over to CALC, and scroll down to **C: SinReg**. The newer TI-84s will show a screen like the one shown here. Up the iterations to 10. Select the lists where your data is stored (X=time). Fill in the period you found in M. Store the regression equation in Y2 by selecting 

, and then Y-VARS, FUNCTION, and Y2. Finally, select Calculate.

On older calculators without the Stats Wizard, your screen will look like the last line shown. Specify the lists and the function to store the equation if they are not the defaults L_1 and L_2. You should see output that looks similar to this:



Compare your results to those obtained from the calculator. Graph both functions. Which one appears to be a better fit? Why?

Check with your instructor.

S. Now measure the amplitude of your wave at each highest point and each lowest point of each cycle of the graph. Record the time of each peak or valley, and the distance from the equilibrium. Find the distance by calculating $equilibrium - \min[y(t)]$ at the valleys, and $\max[y(t)] - equilibrium$ at the peaks.

time	Distance from equilibrium

T. Store the time data in one of the lists in the calculator (one you aren't using), and the distance data in another list. Record the lists you used here. [To enter the data, press STAT and Edit.]

U. Graph the data. What kind of function does this appear to trace out?

V. To model this function, use one of the regression functions under STAT, CALC. Which model fits the data the best? Record your sample models and r^2 values here.

Regression model type	**Regression Equation**	r^2 **or** R^2	**Does the equation fit properties of the data (particularly end behavior)?**

Check with your instructor.

W. As we know from solving models in class, we should obtain a solution of the form $y(t) = Ae^{rt}\sin(\omega t + \delta) + C$. Do you obtain a function of this form if you combine the information from R and V?

X. Graph the equation above against your spring-mass data from the motion detector. How well does it fit the data?

Check with your instructor.

Y. What variables do you think affect the frequency of the spring-mass system?

Z. Design an experiment to test how frequency depends on the mass of the system. Make sure you identify any independent, dependent and controlled variables. Describe how you will obtain the frequency from the graphs you get.

AA. Determine what values of the mass you will use from the available materials (measured in BBs). You will need to get at least 6 data points. Fill in the table below.

<table>
<tr><td colspan="2">State Experimental Question</td></tr>
<tr><td colspan="2">Controlled variables and their units are</td></tr>
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Units are</td><td>Dependent variable is

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BB. Carry out your experiment. Record your data in the above table.

CC. Plot your data and sketch the graph below.

DD. Find the best-fit regression equation. You will need to try several equations. Graph each one with the data to see if it matches the data's pattern, for both large and small x. Keep track of what equations you tried and the r^2 values, etc., in the table below.

Regression model type	**Regression Equation**	$\boldsymbol{r^2 or R^2}$	**Does the equation fit properties of the data (particularly end behavior)?**

Check with your instructor.

EE. Write the best-fit regression equation below along with your reasons for choosing it. Instead of using and x, y, use f for frequency and m for mass.

FF. Can you improve the model you chose by transforming your variables? Why or why not?

Check with your instructor.

GG. Given how frequency depends on mass in the equation above, how should the period depend on the mass? Write an equation for the period T as a function of mass below. Explain your reasoning.

Check with your instructor.

HH. Suppose that the mass of the system is quadrupled. What would happen to the frequency of the system? What would happen to the period of the system? Show your work.

II. Suppose the mass of the system was reduced by a factor of $\frac{1}{64}$. What would happen to the frequency of the system? What would happen to the period of the system? Show your work.

JJ. Suppose that the period of your system is 0.16 seconds. Find the frequency of the system and the mass of the system. Show your work.

KK. Solve the differential equation $my'' + \gamma y' + ky = 0$ symbolically. How does the frequency depend on mass in your solution? Does it agree with your answers in W and EE/FF?

Check with your instructor.

LL. Can you determine the spring constant and damping constant of your spring using the information obtained from your equations?