MAT 212, Quiz #8, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Reindex the series $\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$ to start at $n = 0$.

$$\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2}\, x^n$$

2. Find the first and second derivative of $y = \sum_{n=0}^{\infty} a_n x^n$.

$$y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$$

3. Find the radius of convergence of

a. $\sum_{n=1}^{\infty} \frac{(-1)^n n^2 (x+2)^n}{3^n}$

$$\lim_{n\to\infty} \left| \frac{\cancel{(-1)^{n+1}}(n+1)^2 (x+2)^{n+1}}{3^{n+1}} \cdot \frac{3^n}{\cancel{(-1)^n}(n)^2 (x+2)^n} \right|$$

$$\lim_{n\to\infty} \left(\frac{(n+1)^2}{n^2} \cdot \frac{x+2}{3} \right) < 1 \qquad \left|\frac{x+2}{3}\right| < 1$$

$$-1\cdot 3 - 2 < x+2 - 2 < 1\cdot 3 - 2$$

$$-5 < x < 1$$

radius of convergence is 3

b. $\sum_{n=1}^{\infty} \frac{n! x^n}{n^n}$

$$\lim_{n\to\infty} \left| \frac{(n+1)!\, x^{n+1}}{(n+1)^{n+1}} \cdot \frac{n^n}{n!\, x^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(n+1)}\, x\, n^n}{(n+1)^{n+\cancel{1}}} \right| = \lim_{n\to\infty} \frac{n^n}{(n+1)^n} \cdot x$$

$$\left(\frac{n}{n+1}\right)^n = \left(1 + \frac{1}{n+1}\right)^n \to e^{-1}$$

$$\left|\frac{x}{e}\right| < 1 \Rightarrow |x| < e \text{ radius of convergence}$$

4. Write each expression as a series with the given center.

a. $e^x, x_0 = 0$

$$1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \dots \qquad \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

b. $x^2, x_0 = 1$

$$(x-1)^2 = x^2 - 2x + 1$$

$$\boxed{(x-1)^2 + 2(x-1) + 1} \qquad 2(x-1) = 2x - 2$$

$$x^2 - 1$$