MAT 212, Quiz #6, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Use reduction of order to solve $t^2y'' - t(t+2)y' + (t+2)y = 0, y_1 = t$.

$y_2 = v(t)t$
$y_2' = v't + v$
$y_2'' = v''t + 2v'$

$t^2(v''t + 2v') - t(t+2)(v't + v) + (t+2)(vt) = 0$

$t^3v'' + \cancel{2t^2v'} - t^3v' - \cancel{2t^2v'} - \cancel{t^2v} - \cancel{2tv} + \cancel{t^2v} + \cancel{2vt} = 0$

$\dfrac{t^3v'' + (-t^3)v' = 0}{t^3}$

$v'' - v' = 0$ let $u = v'$, $u' = v''$

$u' - u = 0$

$u' = u$

$\frac{du}{dt} = u \Rightarrow \int \frac{du}{u} = \int dt$

$\ln u = t + c$

$u = e^{t+c} \Rightarrow Ae^t$

$v' = Ae^t$

$v = \int Ae^t\, dt = Ae^t$

$\boxed{y_2 = e^t}$

2. Use the method of undetermined coefficients to solve $y'' + 2y' + y = 2e^{-t}$.

$r^2 + 2r + 1 = 0$ $(r+1)^2 = 0$ $r = -1$

$y(t) = c_1 e^{-t} + c_2 t e^{-t}$

$Y(t) = At^2e^{-t}$

$Y'(t) = A2te^{-t} - At^2e^{-t}$

$Y''(t) = 2Ae^{-t} - 2Ate^{-t} - 2Ate^{-t} + At^2e^{-t}$

$= 2Ae^{-t} - 4Ate^{-t} + At^2e^{-t}$

$2Ae^{-t} - \cancel{4Ate^{-t}} + \cancel{At^2e^{-t}} + \cancel{4Ate^{-t}} - \cancel{2At^2e^{-t}} + \cancel{At^2e^{-t}} = 2e^{-t}$

$2Ae^{-t} = 2e^{-t} \Rightarrow A = 1$

$\boxed{y_p(t) = c_1 e^{-t} + c_2 te^{-t} + t^2e^{-t}}$

3. Find the inverse of $A = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$.

$\det A = 2 + 3 = 5$

$\frac{1}{5}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2/5 & -3/5 \\ 1/5 & 1/5 \end{pmatrix}$

4. What Ansatz would you need to solve for the given forcing function $F(t)$ and the specified solutions $y_1(t)$, $y_2(t)$ to the second order ODE.

	$y_1(t)$	$y_2(t)$	$F(t)$	Ansatz
a.	$\sin t$	$\cos t$	$3e^{2t}$	Ae^{2t}
b.	e^{-t}	e^{-4t}	$-5e^t \cos 2t$	$Ae^t \cos 2t + Be^t \sin 2t$
c.	e^t	e^{-2t}	$t^2 + 7e^t$	$At^2 + Bt + C + Dte^t$
d.	$\sin 3t$	$\cos 3t$	$4 \sin 3t$	$At \sin 3t + Bt \cos 3t$

5. A spring with a mass of 2 kg has damping constant 14, and a force of 6 N is required to keep the spring stretched 2 m beyond its natural length. The spring is stretched 1 m beyond its natural length and then released with zero velocity. Find the position of the mass at any time t. (Set up the ODE and state initial conditions only; you don't need to solve.)

$m = 2 \quad \gamma = 14$

$F = kx$

$6 = k2$

$k = 3$

$\boxed{\begin{array}{l} y(0) = 1 \\ y'(0) = 0 \end{array}}$

$my'' + \gamma y' + ky = 0$

$\boxed{2y'' + 14y' + 3y = 0}$