MAT 212, Exam #3, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Rewrite $x^2 - 2$ as power series centered at $x_0 = 1$. (4 points)

$(x-1)^2$ $\quad x^2-2x+1$

$2(x-1)$ $\quad 2x-2$

-1 $\quad -1$

$x^2 \quad -2$

$\boxed{(x-1)^2 + 2(x-1) - 1}$

or taylor

$(x^2-2)(1) = (1^2-2) = -1$

$(x^2-2)'(1) = (2x)(1) = 2(1) = 2$

$(x^2-2)''(1) = 2(1) = 2$

$-1 + \frac{2}{1!}(x-1) + \frac{2}{2!}(x-1)^2$

2. Find the first and second derivatives of $y = \sum_{n=0}^{\infty} a_n (x-2)^n$. (6 points)

$$y' = \sum_{n=1}^{\infty} n a_n (x-2)^{n-1}$$

$$y'' = \sum_{n=2}^{\infty} n(n-1) a_n (x-2)^{n-2}$$

3. Rewrite each series so that the power term is x^n. (4 points each)

a. $\sum_{n=2}^{\infty} n(n-1) a_n x^{n-1}$

$$\sum_{n=1}^{\infty} (n+1)n\, a_{n+1} x^n$$

b. $\sum_{n=0}^{\infty} a_n x^{n+2}$

$$\sum_{n=2}^{\infty} a_{n-2} x^n$$

4. Rewrite $2x \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + 3 \sum_{n=0}^{\infty} a_n x^n$ as a single sum involving x^n. (6 points)

$$\sum_{n=2}^{\infty} 2\, n(n-1) a_n x^{n-1} + \sum_{n=0}^{\infty} 3a_n x^n$$

$$\sum_{n=1}^{\infty} 2(n+1)n\, a_{n+1} x^n + \sum_{n=0}^{\infty} 3a_n x^n$$

$$3a_0 + \sum_{n=1}^{\infty} 3a_n x^n$$

$$3a_0 + \sum_{n=1}^{\infty} \left[2n(n+1) a_{n+1} + 3a_n\right] x^n$$

5. Use power series to solve $y'' + xy' + 2y = 0$ centered at $x_0 = 0$. Write out at least 4 terms of each solution. (12 points)

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + x\sum_{n=1}^{\infty} na_n x^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=1}^{\infty} na_n x^n + \sum_{n=0}^{\infty} 2a_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1)a_{n+2} x^n + \sum_{n=1}^{\infty} na_n x^n + \sum_{n=0}^{\infty} 2a_n x^n = 0$$

$$2a_2 + 2a_0 + \sum_{n=1}^{\infty} (n+2)(n+1)a_{n+2} x^n + \sum_{n=1}^{\infty} na_n x^n + \sum_{n=1}^{\infty} 2a_n x^n = 0$$

$$2a_2 + 2a_0 + \sum_{n=1}^{\infty} \left[(n+2)(n+1)a_{n+2} + na_n + 2a_n\right] x^n = 0$$

$2a_2 + 2a_0 = 0$

$a_2 = -a_0$

$$(n+2)(n+1)a_{n+2} + (n+2)a_n = 0$$

$$a_{n+2} = \frac{-\cancel{(n+2)}a_n}{\cancel{(n+2)}(n+1)} = \frac{-a_n}{n+1}$$

$n=1$ $\quad a_3 = \frac{-a_1}{2} = -\frac{1}{2}a_1$

$n=2$ $\quad a_4 = \frac{-a_2}{3} = -\frac{1}{3}(-a_0) = \frac{1}{3}a_0$

$n=3$ $\quad a_5 = \frac{-a_3}{4} = -\frac{1}{4}\left(-\frac{1}{2}a_1\right) = \frac{1}{8}a_1$

$n=4$ $\quad a_6 = \frac{-a_4}{5} = -\frac{1}{5}\left(\frac{1}{3}a_0\right) = -\frac{1}{15}a_0$

$n=5$ $\quad a_7 = \frac{-a_5}{6} = -\frac{1}{6}\left(\frac{1}{8}\right)a_1 = \frac{-1}{48}a_1$

$n=6$ $\quad a_8 = \frac{-a_6}{7} = -\frac{1}{7}\left(\frac{-1}{15}\right)a_0 = \frac{1}{105}a_0$

$n=7$ $\quad a_9 = \frac{-a_7}{8} = -\frac{1}{8}\left(\frac{1}{48}a_1\right) = \frac{+1}{384}a_1$

$$y(x) = a_0\left(1 - x^2 + \frac{1}{3}x^4 - \frac{1}{15}x^6 + \frac{1}{105}x^8 + \dots\right) + a_1\left(x - \frac{1}{2}x^3 + \frac{1}{8}x^5 - \frac{1}{48}x^7 + \frac{1}{384}x^9 + \dots\right)$$

6. Rewrite the equation $x^2y'' + (x+1)y' + 3y = 0$ so that it can be solved with a series centered at $x_0 = 2$. Proceed to solve the system only to the point where the equation can be written in a single summation. (8 points)

$X^2 = (x-2)^2 = x^2-4x+4$

$4(x-2) \quad 4x-8$

$4 \quad +4$

x^2

$[(x-2)^2+4(x-2)+4]y'' + [(x-2)+3]y' + 3y = 0$

$\sum_{n=2}^{\infty} n(n-1)a_n(x-2)^{n-2} \quad \sum_{n=1}^{\infty} na_n(x-2)^{n-1} \quad \sum_{n=0}^{\infty} a_n(x-2)^n$

$X+1 = x-2$

$+3$

$\sum_{n=2}^{\infty} n(n-1)a_n(x-2)^n + \sum_{n=2}^{\infty} 4n(n-1)a_n(x-2)^{n-1} + \sum_{n=2}^{\infty} 4n(n-1)a_n(x-2)^{n-2}$

$+ \sum_{n=1}^{\infty} na_n(x-2)^n + \sum_{n=1}^{\infty} 3na_n(x-2)^{n-1} + \sum_{n=0}^{\infty} 3a_n(x-2)^n = 0$

$\sum_{n=2}^{\infty} n(n-1)a_n(x-2)^n + \sum_{n=1}^{\infty} 4(n+1)na_{n+1}(x-2)^n + \sum_{n=0}^{\infty} 4(n+2)(n+1)a_{n+2}(x-2)^n + \sum_{n=0}^{\infty} 3(n+1)a_{n+1}(x-2)^n + \sum_{n=1}^{\infty} na_n(x-2)^n$

$+ \sum_{n=0}^{\infty} 3a_n(x-2)^n = 0$

$\boxed{\begin{gathered}4(2)(1)a_2(x-2) + 4(2)(1)a_2(1) + 4(3)(2)a_3(x-2) + 3(1)a_1 + 3(2)a_2(x-2) + 1a_1(x-2) + 3a_0 + 3a_1(x-2) + \\ \sum_{n=2}^{\infty}\left[n(n-1)a_n + 4(n+1)na_{n+1} + 4(n+2)(n+1)a_{n+2} + 3(n+1)a_{n+1} + na_n + 3a_n\right](x-2)^n = 0\end{gathered}}$

7. For each equation, determine the location of any singular points, and for each one, classify it as regular or irregular. (4 points each)

a. $x^2(1-x)^2y'' + 2xy' + 4y = 0$

$\frac{2x}{x^2(1-x)^2} = \frac{2}{x(1-x)^2} \quad \frac{4}{x^2(1-x)^2}$

$x=0$ regular $\quad x=1$ irregular

b. $x(x-3)y'' + (x+1)y' - 2y = 0$

$\frac{x+1}{x(x-3)} \quad \frac{2}{x(x-3)}$

$x=0, x=3$ both are regular

c. $(\sin x)y'' + xy' + 4y = 0$

$\frac{x}{\sin x} \quad \frac{4}{\sin x} \quad \lim_{x\to 0} \frac{x}{\sin x} = 1 \quad x=0$ regular $\lim_{x\to 0}\frac{x^2}{\sin x} = 0$

$x=0$ regular

$x \lim_{x\to n\pi} \frac{(x-n\pi)}{\sin x} = \frac{1\cdot x}{\cos x} = 1(n\pi)$

$\lim \frac{(x-n\pi)^2}{} = \frac{2(x-n\pi)}{\cos x} = 0$

8. The equation $xy'' + y = 0$ has a regular singular point at $x = 0$. Use that fact to find two solutions to the system. Find the first 4 terms of each solution. (12 points)

$$y=\sum_{n=0}^{\infty} a_n x^{r+n} \quad y'=\sum_{n=1}^{\infty}(r+n)a_n x^{r+n-1} \quad y''=\sum_{n=2}^{\infty}(r+n-1)(r+n)a_n x^{r+n-2}$$

$$x\sum_{n=2}^{\infty}(r+n-1)(r+n)a_n x^{r+n-2} + \sum_{n=0}^{\infty} a_n x^{r+n}$$

$$\sum_{n=2}^{\infty}(r+n-1)(r+n)a_n x^{r+n-1} + \sum_{n=0}^{\infty} a_n x^{r+n} = 0$$

$$\sum_{n=1}^{\infty}(r+n)(r+n+1)a_{n+1} x^{r+n} + \sum_{n=0}^{\infty} a_n x^{r+n} = 0$$

$$\sum_{n=1}^{\infty}\left[(r+n)(r+n+1)a_{n+1} + a_n\right]x^{r+n} + a_0 x^r = 0$$

$$a_0 = 0$$

$$(r+n)(r+n+1)a_{n+1} + a_n = 0$$

$$a_{n+1} = \frac{-a_n}{(r+n)(r+n+1)} \qquad \text{no condition on } r$$

$$[(x-1)+1]y'' + y = 0 \qquad x_0 = 1$$

$$(x-1)\sum_{n=2}^{\infty} n(n-1)a_n(x-1)^{n-2} + \sum_{n=2}^{\infty} n(n-1)a_n(x-1)^{n-2} + \sum_{n=0}^{\infty} a_n(x-1)^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1)a_n(x-1)^{n-1} + \sum_{n=2}^{\infty} n(n-1)a_n(x-1)^{n-2} + \sum_{n=0}^{\infty} a_n(x-1)^n = 0$$

$$\sum_{n=1}^{\infty}(n+1)n\,a_{n+1}(x-1)^n + \sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}(x-1)^n + \sum_{n=0}^{\infty} a_n(x-1)^n = 0$$

$$\sum_{n=1}^{\infty}\left[(n+1)n\,a_{n+1} + (n+2)(n+1)a_{n+2} + a_n\right](x-1)^n + 2(1)a_2\cancel{(x-1)}^0 + a_0\cancel{(x-1)}^0 = 0$$

$$2a_2 + a_0 = 0$$

$$a_2 = -\tfrac{1}{2}a_0$$

$$a_{n+2} = \frac{-a_n - n(n+1)a_{n+1}}{(n+2)(n+1)} = -\frac{a_n}{(n+2)(n+1)} - \frac{n\,a_{n+1}}{n+2}$$

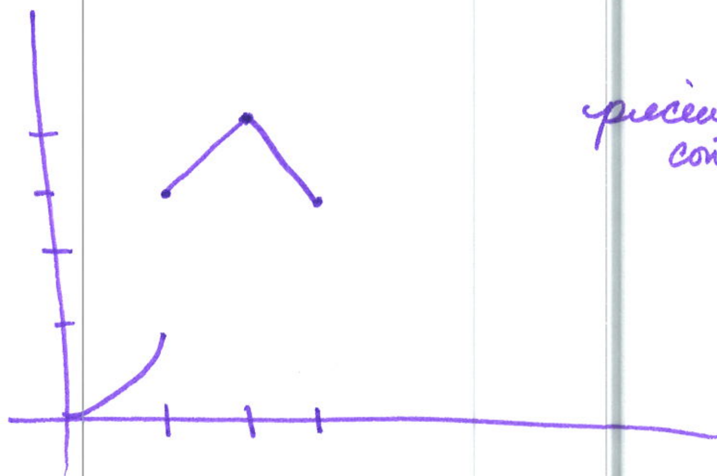
$n=1$ $$a_3 = \frac{-a_1}{3\cdot 2} - \frac{1\cdot a_2}{3} = -\tfrac{1}{6}a_1 - \tfrac{1}{3}\left(-\tfrac{1}{2}a_0\right) = -\tfrac{1}{6}a_1 + \tfrac{1}{6}a_0$$

$n=2$ $$a_4 = \frac{-a_2}{4\cdot 3} - \frac{2a_3}{4} = -\tfrac{1}{12}\left(-\tfrac{1}{2}a_0\right) - \tfrac{1}{2}\left(-\tfrac{1}{6}a_1 + \tfrac{1}{6}a_0\right) = \tfrac{1}{24}a_0 + \tfrac{1}{12}a_1 - \tfrac{1}{12}a_0 = -\tfrac{1}{24}a_0 + \tfrac{1}{12}a_1$$

$n=3$ $$a_5 = \frac{-a_3}{5\cdot 4} - \frac{3a_4}{5} = -\tfrac{1}{20}\left(-\tfrac{1}{6}a_1 + \tfrac{1}{6}a_0\right) - \tfrac{3}{5}\left(-\tfrac{1}{24}a_0 + \tfrac{1}{12}a_1\right) = \tfrac{1}{120}a_1 - \tfrac{1}{120}a_0 + \tfrac{1}{40}a_0 - \tfrac{1}{20}a_1 = -\tfrac{1}{24}a_1 + \tfrac{1}{60}a_0$$

$$y(x) = a_0\left(1 - \tfrac{1}{2}x^2 + \tfrac{1}{6}x^3 - \tfrac{1}{24}x^4 + \tfrac{1}{60}x^5 + \dots\right) + a_1\left(x - \tfrac{1}{6}x^3 + \tfrac{1}{12}x^4 - \tfrac{1}{24}x^5 + \dots\right)$$

9. Graph the function $f(t) = \begin{cases} t^2, & 0 \le t < 1 \\ 2+t, & 1 \le t < 2. \\ 6-t, & 2 \le t < 3 \end{cases}$ Is the function continuous, piecewise continuous, or neither? (6 points)



piecewise continuous

10. Find the Laplace transform using the definition for the function $f(t) = \sinh t$. (8 points)

$$\int_0^\infty e^{-st}\left(\tfrac12 e^{t} - \tfrac12 e^{-t}\right)dt = \frac12\int_0^\infty e^{-st+t} - e^{-st-t}\,dt \qquad \sinh t = \tfrac12(e^{t}-e^{-t})$$

$$\frac12\int_0^\infty e^{-t(s-1)} - e^{-t(s+1)}\,dt = -\frac12\left[\frac{e^{-t(s-1)}}{s-1} + \frac{e^{-t(s+1)}}{s+1}\right]_0^\infty =$$

$$\frac{-1}{2}\left[\frac{-(s+1)e^{-t(s-1)} + e^{-t(s+1)}(s-1)}{s^2-1}\right]_0^\infty = \frac{-1}{2(s^2-1)}\left[0+0+(s+1)e^0-(s-1)e^0\right] =$$

$$\frac{-1(\cancel{s}+1-\cancel{s}+1)}{-2(s^2-1)} = \frac{-2}{-2(s^2-1)} = \boxed{\frac{1}{s^2-1}}$$

11. Use the attached table to find the inverse Laplace transform of each function. (5 points each)

a. $F(s) = \frac{8s^2-4s+12}{s(s^2+4)}$

$\frac{A}{s} + \frac{Bs+C}{s^2+4} = As^2+4A+Bs^2+Cs = 8s^2-4s+12$ $\quad -2(2)$

$4A=12 \quad A=3$

$C\cancel{s}=-4\cancel{s} \quad C=-4$

$3\cancel{s^2}+B\cancel{s^2}=8\cancel{s^2}$

$B=5$

$\frac{3}{s} + \frac{5s}{s^2+4} + \frac{-4}{s^2+4}$

$\boxed{3+5\cos 2t - 2\sin 2t}$

c. $F(s) = \frac{(s-2)e^{-3}}{s^2-4s+3}$

$(s-3)(s-1) \qquad \frac{A}{s-3} + \frac{B}{s-1} \qquad As-A+Bs-3B$

$A+B=1$

$-A-3B=-2$

$A=B=\frac12$

$\frac12 e^{-3}\left(\frac{1}{s-3}+\frac{1}{s-1}\right)$

$\frac12 e^{-3}\left(e^{3t}+e^{t}\right) = \frac12\left[e^{3(t-1)}+e^{t-3}\right]$

b. $F(s) = \frac{1}{s^4(s^2+1)}$

$\cancel{\frac{A}{s}} + \frac{B}{s^2} + \cancel{\frac{C}{s^3}} + \frac{D}{s^4} + \frac{\cancel{Es}+F}{s^2+1}$

$As^3(s^2+1)+Bs^2(s^2+1)+Cs(s^2+1)+D(s^2+1)+(Es+F)s^4 =$

$As^5+As^3+Bs^4+Bs^2+Cs^3+Cs+Ds^2+D+Es^5+Fs^4 = 1$

$A+E=0$ ✓ $\quad C=0$

$B+F=0 \quad A=0$

$A+C=0$ ✓ $\quad E=0$

$B+D=0$ ✓ $\quad D=1$

$C=0$ ✓ $\quad B=-1$

$D=1$ ✓ $\quad F=1$

$\frac{-1}{s^2} + \frac{1}{s^4} + \frac{1}{s^2+1} \Rightarrow \boxed{-t+\frac16 t^3+\sin t}$

or $\boxed{\int_0^t \frac16 (t-\tau)^3 \sin\tau\, d\tau}$

12. Rewrite $f(t) = \begin{cases} t, 0 \le t < 2 \\ 2, \quad t \ge 2 \end{cases}$ in terms of the unit step function. (6 points)

$t - u_2(\cancel{4}^{\,2-t}-t)$

or

$t + (2-t)u(t-2)$

13. Find the Laplace transform of $f(t) = \int_0^t (t-\tau)e^{\tau}d\tau$. (5 points)

$t * e^t$

$\frac{1}{s^2} \cdot \frac{1}{s-1} = \frac{1}{s^2(s-1)}$

14. For each of the equations, find the Laplace transform. Stop solving when you are about to do the inverse Laplace transform (i.e. fully simplify). (6 points each)

a. $y'' + 4y = u_\pi(t) - u_{3\pi}(t), y(0) = 0, y'(0) = 0$

$s^2F(s) - 0 - 0 + 4F(s) = \frac{e^{-\pi s}}{s} - \frac{e^{-3\pi s}}{s}$

$F(s) \cdot (s^2+4) = \frac{1}{s}(e^{-\pi s} - e^{-3\pi s}) \Rightarrow F(s) = \frac{1}{s(s^2+4)}(e^{-\pi s} - e^{-3\pi s})$

b. $y^{IV} - y = \delta(t-1), y(0) = 0, y'(0) = 0, y''(0) = 0, y'''(0) = 0$

$s^4F(s) - F(s) = e^{-s}$

$F(s)(s^4-1) = e^{-s} \Rightarrow F(s) = \frac{e^{-s}}{s^4-1} = \frac{e^{-s}}{(s^2+1)(s^2-1)} = \frac{e^{-s}}{(s^2+1)(s-1)(s+1)}$

15. Use a Laplace transform to (fully) solve $y'' - 2y' + 4y = 0, y(0) = 2, y'(0) = 0$. (12 points)

$$s^2F(s) - s(2) - 0 - 2(sF(s) - 2) + 4F(s) = 0$$

$$F(s)(s^2 - 2s + 4) - 2s + 4 = 0$$

$$F(s) = \frac{2s-4}{s^2-2s+4} = \frac{2s-4}{(s^2-2s+1)+3} = \frac{2s-4}{(s-1)^2+3} = \frac{2s}{(s-1)^2+3} \quad \frac{-4}{(s-1)^2+3}$$

$$\frac{2(s-1)}{(s-1)^2+3} \quad \frac{-2}{(s-1)^2+3}$$

$$y(t) = 2e^t \cos(\sqrt{3}t) - \frac{2e^t}{\sqrt{3}} \sin(\sqrt{3}t)$$

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

Laplace transforms – Table			
$f(t) = L^{-1}\{F(s)\}$	$F(s)$	$f(t) = L^{-1}\{F(s)\}$	$F(s)$
$a \quad t \geq 0$	$\frac{a}{s} \quad s > 0$	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
$at \quad t \geq 0$	$\frac{a}{s^2}$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
e^{-at}	$\frac{1}{s+a}$	$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$
te^{-at}	$\frac{1}{(s+a)^2}$	$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2 + \omega^2}$
$\frac{1}{2}t^2 e^{-at}$	$\frac{1}{(s+a)^3}$	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}$
$\frac{1}{(n-1)!}t^{n-1}e^{-at}$	$\frac{1}{(s+a)^n}$	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
e^{at}	$\frac{1}{s-a} \quad s > a$	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$
te^{at}	$\frac{1}{(s-a)^2}$	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2} \quad s > \lvert\omega\rvert$
$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$\frac{1}{a^2}[1 - e^{-at}(1 + at)]$	$\frac{1}{s(s+a)^2}$	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
t^n	$\frac{n!}{s^{n+1}} \quad n = 1,2,3..$	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}} \quad s > a$	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}} \quad s > a$	$1 - e^{-at}$	$\frac{a}{s(s+a)}$
$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{3/2}}$	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}} \quad s > 0$	$f(t - t_1)$	$e^{-t_1 s}F(s)$
$g(t) \cdot p(t)$	$G(s) \cdot P(s)$	$f_1(t) \pm f_2(t)$	$F_1(s) \pm F_2(s)$
$\int f(t)dt$	$\frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$	$\delta(t)$ *unit impulse*	$1 \quad$ *all s*
$\frac{df}{dt}$	$sF(s) - f(0)$	$\frac{d^2f}{df^2}$	$s^2F(s) - sf(0) - f'(0)$
$\frac{d^nf}{dt^n}$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - s^{n-3}f''(0) - \cdots - f^{n-1}(0)$		