MAT 201, Quiz #8, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ using Stokes' Theorem for $\vec{F}(x, y, z) = yz\hat{\imath} + 2xz\hat{\jmath} + e^{xy}\hat{k}$ over the curve oriented counterclockwise bounded by the intersection of $x^2 + y^2 = 16$ and $z = 5$.

Circle of radius 4 at height 5

$\vec{n} = \hat{k}$

$$\vec{\nabla}\times\vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & 2xz & e^{xy} \end{vmatrix} = (xe^{xy}-2x)\hat{\imath} - (ye^{xy}-y)\hat{\jmath} + (2z-z)\hat{k} \quad z=5$$

$$\left((xe^{xy}-2x)\hat{\imath} + (y-ye^{xy})\hat{\jmath} + 5\hat{k}\right)\cdot\hat{k} = 5$$

$$\int_0^{2\pi}\int_0^4 5r\,dr\,d\theta = 5\cdot\pi(4)^2 = \boxed{80\pi}$$

2. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ for $\vec{F}(x, y, z) = yz\hat{\imath} + xz\hat{\jmath} + (xy + z)\hat{k}$ over the path from $(2,0,-3)$ to $(5,7,-1)$.

$\int yz\,dx = xyz + \text{—}$ $\int xz\,dy = xyz + \text{—}$

$\int (xy+z)\,dz = xyz + \frac{1}{2}z^2 + \text{—}$ conservative

$f = xyz + \frac{1}{2}z^2 + k$

$$f(5,7,-1) - f(2,0,-3) =$$

$$5\cdot7(-1) + \tfrac{1}{2}(-1)^2 - \left(0 + \tfrac{1}{2}(-3)^2\right) = -35 + \tfrac{1}{2} - \tfrac{9}{2} = -35 - 4 = -39$$

3. Evaluate $\oint_C xy\,dx + x^2\,dy$ over the circle centered at the origin with a radius of 3.

$$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 2x - x = x$$

$$\int_0^{2\pi}\int_0^3 r\cos\theta\, r\,dr\,d\theta = \int_0^{2\pi}\int_0^3 r^2\cos\theta\,dr\,d\theta =$$

$$\int_0^{2\pi} \tfrac{1}{3}r^3\Big|_0^3 \cos\theta\,d\theta = 9\int_0^{2\pi}\cos\theta\,d\theta = 9 \quad \sin\theta\Big|_0^{2\pi} = \boxed{0}$$