MAT 201, Quiz #15, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Draw at least 5 level curves of the surface $f(x,y) = \frac{y}{x^2+y^2}$.

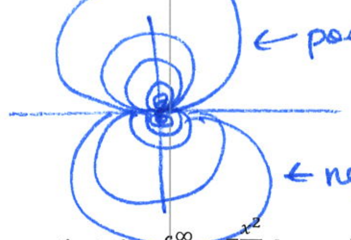
$c = \frac{y}{x^2+y^2} \Rightarrow x^2+y^2 = \frac{1}{c}y$

$r^2 = \frac{1}{c} r\sin\theta$

$r = \frac{1}{c}\sin\theta$

$c = 1, 2, 3, +1/2, 1/3, 1/10$

$-1, -2, -3, -1/2, -1/3, -1/10$



2. Prove that the $\int_{-\infty}^{\infty} e^{-\frac{x^2}{2}}dx = \sqrt{2\pi}$.

$\int_{-\infty}^{\infty} e^{-x^2/2}dx \cdot \int_{-\infty}^{\infty} e^{-y^2/2}dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{-x^2-y^2}{2}}dy\,dx$

$= \int_0^{2\pi}\int_0^{\infty} e^{-r^2/2}\, r\,dr\,d\theta =$ $\quad u = -r^2/2$, $-du = +r\,dr$ $\quad -\int e^u\,du =$

$\int_0^{2\pi} -e^{-r^2/2}\Big|_0^{\infty} d\theta = \int_0^{2\pi} -e^{-\infty}+e^0\,d\theta = \int_0^{2\pi} 1\,d\theta = 2\pi$

but $\int_{-\infty}^{\infty} e^{-x^2/2}dx \cdot \int_{-\infty}^{\infty} e^{-y^2/2}dy = \left(\int_{-\infty}^{\infty} e^{-x^2/2}dx\right)^2$ so $\int_{-\infty}^{\infty} e^{-x^2/2}dx = \sqrt{2\pi}$

3. Find $\frac{\partial z}{\partial y}$ for $yz + x\ln y = z^2$.

$f = yz + x\ln y - z^2 = 0$

$\frac{\partial f}{\partial y} = z + \frac{x}{y}$

$\frac{\partial f}{\partial z} = y - 2z$

$\frac{\partial z}{\partial y} = -\frac{(z+\frac{x}{y})}{y-2z} = \frac{z+\frac{x}{y}}{2z-y}\cdot\frac{y}{y} = \boxed{\frac{zy+x}{2yz-y^2}}$

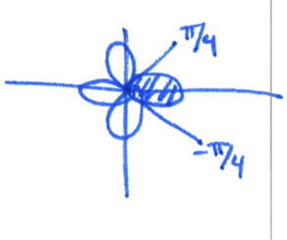
4. Find the area of one petal of $r = \cos 2\theta$ using a double integral.

$\cos 2\theta = 0$

$\cos\alpha = 0$

$\alpha = \pi/2, 3\pi/2, -\pi/2$

$2\theta = \pi/4, 3\pi/4, -\pi/4$



$\int_{-\pi/4}^{\pi/4}\int_0^{\cos 2\theta} r\,dr\,d\theta = \int_{-\pi/4}^{\pi/4} \frac{1}{2}r^2\Big|_0^{\cos 2\theta} d\theta =$

$\frac{1}{2}\int_{-\pi/4}^{\pi/4} \cos^2 2\theta\,d\theta =$ by symmetry $\int_0^{\pi/4}\cos^2 2\theta\,d\theta =$

$\int_0^{\pi/4}\frac{1}{2}(1+\cos 4\theta)\,d\theta = \frac{1}{2}\left(\theta + \frac{1}{4}\sin 4\theta\right)\Big|_0^{\pi/4} =$

$\frac{1}{2}\cdot\frac{\pi}{4} = \boxed{\frac{\pi}{8}}$

5. Evaluate the integral $\iint_R (x+y)e^{x^2-y^2}\,dA$ where R is enclosed by $x-y=0, x-y=2, x+y=2, x+y=3$.

$x^2-y^2=(x+y)(x-y)$

$u=x-y$ $[0,2]$ $v=x+y$ $[2,3]$

$v=x+y$
$u=x-y$
$v+u=2x$ $x=\frac{1}{2}(u+v)$

$u=x-y$
$-v=-x-y$
$\frac{u-v}{-2}=\frac{-2y}{-2}$ $y=-\frac{1}{2}(u-v)$

$J=\begin{vmatrix} 1/2 & 1/2 \\ -1/2 & 1/2 \end{vmatrix}=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

$\int_0^2\int_2^3 \frac{1}{2}ve^{uv}\,dv\,du=$

$\frac{1}{2}\int_2^3\int_0^2 ve^{uv}\,du\,dv=\frac{1}{2}\int_2^3 \frac{\cancel{v}e^{uv}}{\cancel{v}}\Big|_0^2\,dv=$

$\frac{1}{2}\int_2^3 e^{2v}-1\,dv=\frac{1}{2}\left(\frac{1}{2}e^{2v}-v\right|_2^3=$

$\frac{1}{2}\left(\frac{1}{2}e^6-3-\frac{1}{2}e^4+2\right)=\boxed{\frac{1}{4}e^6-\frac{1}{4}e^4-\frac{1}{2}}$

6. Evaluate the line integral $\int_C \rho(x,y)ds$ to find the mass of a wire following the path given by $C:\vec{r}(t)=t^3\hat{\imath}+t\hat{\jmath}, 0\le t\le 2$ with density function $\rho(x,y)=y^3$.

$y=t$ $\rho=t^3$

$r'(t)=3t^2\hat{\imath}+1\hat{\jmath}$ $\|r'(t)\|=\sqrt{9t^4+1}$

$\int_0^2 t^3\sqrt{9t^4+1}\,dt=$

$u=9t^4+1$
$du=36t^3 \Rightarrow \frac{1}{36}du=t^3dt$

$\int u^{1/2}\cdot\frac{1}{36}du$

$\frac{1}{3}\cdot\frac{1}{18}u^{3/2}$

$\frac{1}{54}(9t^4+1)^{3/2}\Big|_0^2=$

$\frac{1}{54}\left[145^{3/2}-1^{3/2}\right]\approx 32.32$