MAT 201, Quiz #10, Spring 2016 Name KEY

Instructions: Show all work. Answers without work required to obtain the solution will not receive full credit. Some questions may contain multiple parts: be sure to answer all of them. Give exact answers unless specifically asked to estimate.

1. Graph the gradient field for the function $f(x, y) = x^2 + xy + y^2 + y$ and locate any extrema. Does the gradient field say the extrema are maxima, minima, saddle points or cannot be determined based on the graph? Graph at least 5 level curves.

$\nabla f = \langle 2x+y,\ x+2y+1 \rangle$

$f_x = 2x+y = 0 \rightarrow y = -2x$

$f_y = x+2y+1 = 0$

$y = -\frac{1}{2}x - \frac{1}{2}$

$-2x = -\frac{1}{2}x - \frac{1}{2}$

$-4x = -x - 1$

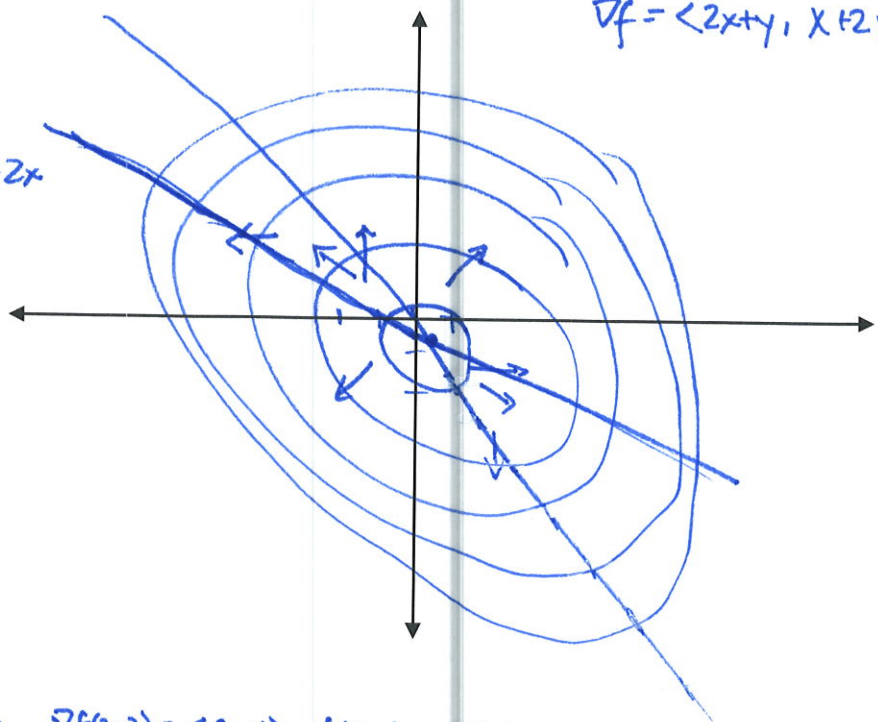
$+x \quad +x$

$-3x = -1$

$\frac{-3x}{-3} = \frac{-1}{-3}$

$x = \frac{1}{3} \quad y = -\frac{2}{3}$

$\boxed{(1/3, -2/3) \text{ min}}$



$\nabla f(1,1) = \langle 3,4 \rangle \quad \nabla f(0,-2) = \langle -2,-3 \rangle \quad \nabla f(2,-2) = \langle 2,-1 \rangle \quad f(-2,1) = \langle -3,1 \rangle$

2. For the same function as in #1, use the second partials test to confirm your results.

$f_{xx} = 2$

$f_{yy} = 2$

$f_{xy} = 1$

$D = 2 \cdot 2 - (1)^2 = 4 - 1 = 3$ min or max

$f_{xx} > 0$ $\boxed{\text{min}}$

3. Find the length of the curve $\vec{r}(t) = \cos t\,\hat{\imath} + \sin t\,\hat{\jmath} + \ln\cos t\,\hat{k}, 0 \le t \le \frac{\pi}{4}$.

$r'(t) = -\sin t\,\hat{\imath} + \cos t\,\hat{\jmath} + -\frac{\sin t}{\cos t}\hat{k}$

$\|r'(t)\| = \sqrt{\cos^2 t + \sin^2 t + \tan^2} = \sqrt{1+\tan^2 t} = \sqrt{\sec^2 t} = \sec t$

$\int_0^{\pi/4} \sec t\, dt = \ln|\sec t + \tan t| \Big|_0^{\pi/4} = \ln|\sqrt{2}+1| - \ln|1+0| =$

$\ln|\sqrt{2}+1|$

4. Find the curvature of $\vec{r}(t) = 3t\hat{\imath} + 4\sin t\,\hat{\jmath} + 4\cos t\,\hat{k}$.

$r'(t) = 3\hat{\imath} + 4\cos t\,\hat{\jmath} + -4\sin t\,\hat{k}$

$r''(t) = 0\hat{\imath} - 4\sin t\,\hat{\jmath} - 4\cos t\,\hat{k}$

$$\frac{\|r'(t)\times r''(t)\|}{\|r'(t)\|^3} = \frac{\sqrt{256+144}}{\left(\sqrt{9+16}\right)^3} = \frac{20}{125}$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & 4\cos t & -4\sin t \\ 0 & -4\sin t & -4\cos t \end{vmatrix} = (-16\cos^2 t - 16\sin^2 t)\hat{\imath} - (-12\cos t)\hat{\jmath} + (-12\sin t)\hat{k}$$

$-16\hat{\imath} + 12\cos t\,\hat{\jmath} - 12\sin t\,\hat{k}$

$$\boxed{\frac{4}{25}}$$

5. Find the surface area of $z = 4 - x^2 - y^2$ above the xy-plane.

$\nabla F = \langle -2x, -2x, -1\rangle$

$\|\nabla F\| = \sqrt{4x^2 + 4y^2 + 1} =$

$$\int_0^{2\pi}\int_0^2 \sqrt{4r^2+1}\; r\,dr\,d\theta =$$

$\frac{1}{8}u^{1/2}\,du$

$u = 4r^2 + 1$

$du = 8r\,dr \rightarrow \frac{1}{8}\,du \quad dr$

$$\int_0^{2\pi} \frac{1}{8}\cdot\frac{2}{3}u^{3/2}\Big|_0^2 d\theta = \int_0^{2\pi}\frac{1}{12}(17^{3/2}-1)\,d\theta = \frac{1}{12}(17^{3/2}-1)2\pi = \frac{\pi}{6}(17^{3/2}-1)$$

6. Find the surface area of $\vec{r}(u,v) = u^2\hat{\imath} + uv\hat{\jmath} + \frac{1}{2}v^2\hat{k}, 0 \le u \le 1, 0 \le v \le 2$

$r_u = 2u\hat{\imath} + v\hat{\jmath} + 0\hat{k}$

$r_v = 0\hat{\imath} + u\hat{\jmath} + v\hat{k}$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2u & v & 0 \\ 0 & u & v \end{vmatrix} = (v^2-0)\hat{\imath} - (2uv-0)\hat{\jmath} + (2u^2-0)\hat{k}$$

$v^2\hat{\imath} - 2uv\hat{\jmath} + 2u^2\hat{k}$

$$\|r_u \times r_v\| = \sqrt{v^4 + 4u^2v^2 + 4u^4} = \sqrt{(v^2+2u^2)^2} = v^2 + 2u^2$$

$$\int_0^1\int_0^2 v^2 + 2u^2\,dv\,du = \int_0^1 \frac{1}{3}v^3 + 2u^2v\Big|_0^2 du = \int_0^1 \frac{8}{3} + 4u^2\,du =$$

$$\frac{8}{3}u + \frac{4}{3}u^3\Big|_0^1 = \frac{8}{3} + \frac{4}{3} = \frac{12}{3} = \boxed{4}$$