MAT 135, Final Exam, Spring 2016 Name KEY

Instructions: Show all work to receive full credit. You should note any formulas used or calculator functions used, their inputs and outputs. I cannot grade work if I don't know where an answer came from. Be sure complete all parts of each questions, including requests for interpretation and explanations. Be as thorough as possible.

1. The 100 tiles in Scrabble are distributed as follows:

Tile	Number	Tile	Number	Tile	Number	Tile	Number
Blank	2	A	9	B	2	C	2
D	4	E	12	F	2	G	3
H	2	I	9	J	1	K	1
L	4	M	2	N	6	O	8
P	2	Q	1	R	6	S	4
T	6	U	4	V	2	W	2
X	1	Y	2	Z	1		

a. What is the probability of selecting an M as the first tile in a game? (5 points)

$$\frac{2}{100} = \frac{1}{50} = 2\%$$

b. What is the probability of selecting a S, T or D as the first tile? (6 points)

$4+6+4=14$

$$\frac{14}{100} = \frac{7}{50} = 14\%$$

c. What is the probability of not selecting a blank? (5 points)

$$1-\frac{2}{100} = \frac{98}{100} = 98\%$$

d. What is the probability of selecting the word CAR in order from the first three tiles in a game? (7 points)

$$\frac{2}{100}\cdot\frac{9}{99}\cdot\frac{6}{98} = 1.11\times10^{-4}$$

e. Is the selection of letters in Scrabble independent? Why or why not? (4 points)

No, Since after selecting a tile there are fewer to choose from & the probability changes

2. Determine the **number** of outcomes in each of the following scenarios. (7 points each)
   a. A local area network requires eight characters for a username and is case sensitive, and the character can use numbers in the last two digits. How many usernames of this type are there?

$52^6 \cdot 62^2 = 7.5998 \times 10^{13}$

   b. Suppose that a lottery has 46 balls, and someone needs 7 matches in any order to win the top prize. How many possible winning number combinations are possible?

$46C7 = 5{,}352{,}680$

   c. Suppose that nine horses are in a particular race. How many ways can the top three horses finish?

$9P3 = 504$

   d. Tim is planning to create a word scramble puzzle out of the word RUMPELSTILTSKIN. How many different sequences of the letters are possible?

$$\frac{15!}{2!2!2!2!} = 8.17 \times 10^{10}$$

3. A charity sells 450 tickets for a raffle, costing $10 per ticket. The top prize is $500, a second prize of $100, and two third prizes of $50. For someone purchasing a ticket, what is the expected value? Interpret the value in the context of the problem. (10 points)

Value	490	90	40	−10
Probability	1/450	1/450	2/450	446/450

$490\left(\frac{1}{450}\right) + 90\left(\frac{1}{450}\right) + 40\left(\frac{2}{450}\right) - 10\left(\frac{446}{450}\right) = -8.44$

for every ticket one buys, one can expect to lose $8.44 on average

4. Four-fifths of drivers put their seat belt on when they get into a driving simulator. Use that fact to answer the following questions. (6 points each)
   a. If 20 people get into the simulator, what is the probability that exactly 8 of the people will put on their seat belts?

$n = 20, p = 4/5$

binomialpdf(20, 4/5, 8) = 8.657×10^{-5}

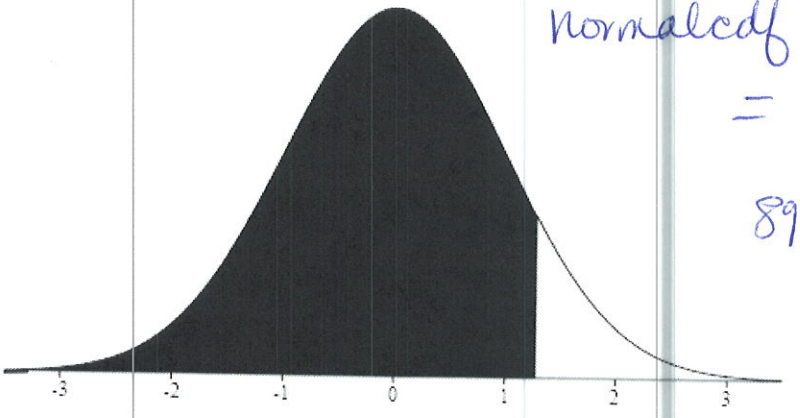
   b. If 20 people get into the simulator, what is the probability that at least ~~than~~ 15 people will put on their seat belts?

. . . 13 14 | 15 16 . . .

1 − binomialcdf(20, 4/5, 14) = .8042

80.4%

5. Find the probability under the curve of the given normal distributions. (6 points each)
   a. Standard normal distribution. Z-score at the boundary is 1.25.



normalcdf(−E99, 1.25)
= .89435

89.4%

6. The SAT has a mean score of 1498 and a standard deviation of 199. (5 points each)
   a. What is the z-score of 1840?

$\frac{1840 - 1498}{199} = 1.71859\ldots$

   b. What score represents the 80th percentile of the distribution? Round your answer to the nearest 10 points.

invNorm(.80, 1498, 199) = 1665.48

1670

c. If a school wants to admit only students with the top 10% of SAT scorers, what cut-off score is needed? Round your answer to the nearest 10 points.

invNorm(.90, 1498, 199) = 1753 ⇒ 1750

d. The mean score on the ACT is 21 with a standard deviation of 5.2. Which student scored higher: Abby with a score of 31 on the ACT, or Barbara with a score of 2130 on the SAT?

$z_A = \frac{31-21}{5.2} = 1.923$

$z_B = \frac{2130-1498}{199} = 3.175\ldots$ Barbara has the higher score

7. For each of the following variables, determine i) is the variable qualitative or quantitative? ii) the level of measurement: nominal, ordinal, interval, or ratio? iii) if the variable is quantitative, is it discrete or continuous? (4 points each)

a. Date of birth

quantitative, discrete interval

b. Credit card brand

qualitative, nominal

c. Social security number

qualitative, nominal

d. Body weight

quantitative, continuous, ratio

8. A particular church choir was asked their ages. The results are shown below in a stemplot below. Key: 3|1 = 31.

Stemplot of ages in church choir

1	5 5 6
2	0 3 3 4 6 6 8
3	1 4 6
4	3 3

For the data above, find the following statistics:

a. The mean, median and mode (6 points)

$\bar{x} = 26.87 \Rightarrow 26.9$

$\tilde{x} = 26$

no mode: 15, 23, 26, 43 all repeated twice

b. The standard deviation and range (6 points)

$S_x = 9.10$

range $= 43 - 15 = 28$

c. Calculate the five-number summary for this data. (5 points)

min = 15

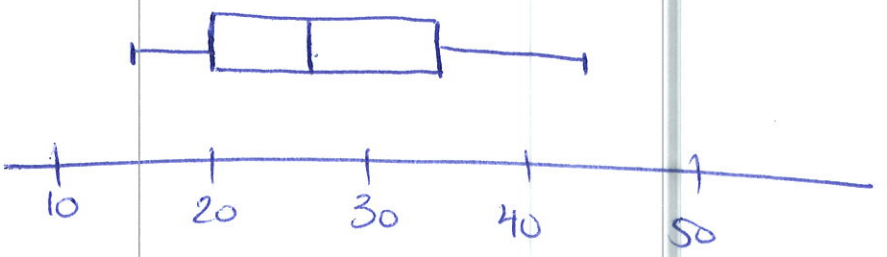
$Q_1 = 20$

Med = 26

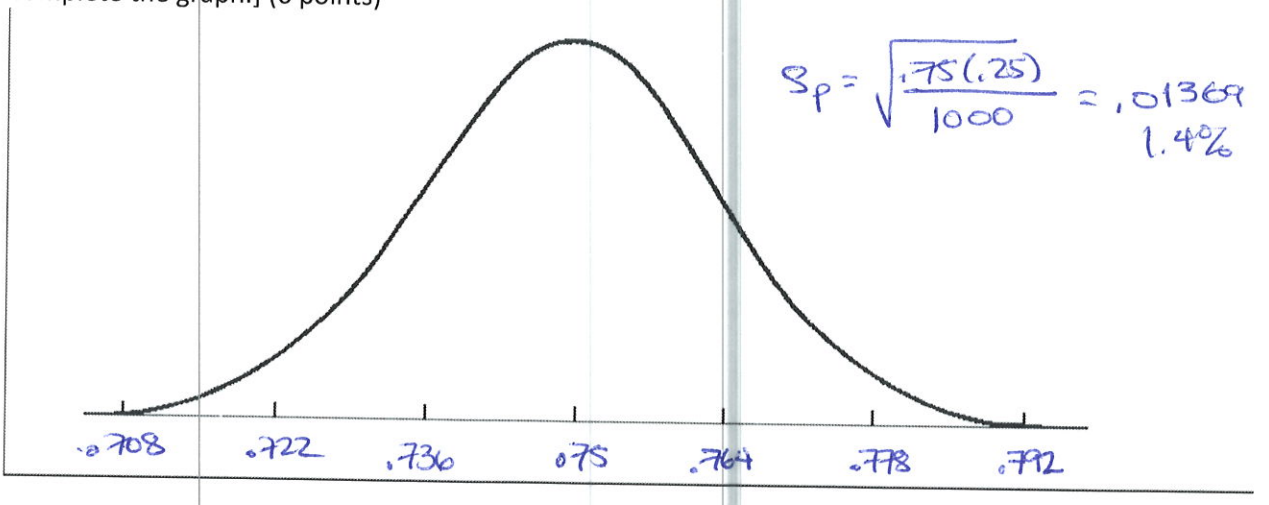
$Q_3 = 34$

Max = 43

d. Use that information to construct a simple box plot. Be sure your graph is to scale. (7 points)



9. If the population proportion is $p = 0.75$. Sketch the sampling distribution on the normal curve below for sample of 1000 people. [Hint: You will need to calculate the standard error to complete the graph.] (6 points)



$s_p = \sqrt{\frac{.75(.25)}{1000}} = .01369$
1.4%

10. A recent poll placed a head-to-head match-up between Hillary Clinton and Donald Trump at 48% Clinton, 41% Trump based on a national poll of 504 people. Construct the 95% confidence interval around both the Clinton and Trump values.

1 PropZInt n=504 Trump
Clinton x=242 C-level: .95 x=207

a. Clinton: (.43654, .52378)

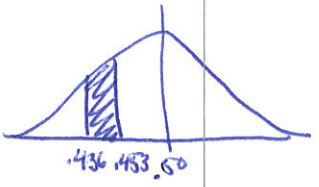
b. Trump: (.36776, .45366)

(8 points)

c. Do the two confidence intervals overlap? If so, in what range of values? (5 points)

yes (.43654 to .45366)

d. If we assume conservatively that the true population proportion is close to 50%, what is the probability that Clinton and Trump are actually tied or their positions are reversed? [Hint: find the probability that the values are normally distributed with a mean of 50%, and falls between the two values in part c. If there is no overlap, report "less than 5%".] (4 points)



$s_p = \sqrt{\frac{.5 \cdot .5}{504}} = .022$

normalcdf(.43654, .45366, .50, .022) = .0156

the area of overlap is so small that they are almost certainly not tied or switched places

11. A Gallup poll taken in 1978 asked 1006 Americans how many books they read in the past year. They found a mean of 18.8 with a standard deviation of 19.8 books. [Note: This results suggests the distribution is very skewed.] Calculate a 99% confidence interval for these results. (points)

T-Interval (Stats)
$\bar{x} = 18.8$
$Sx = 19.8$
$n = 1006$
C-level: .99

(17.189, 20.411)

use T-distribution since the results are very skewed

12. Complete the table below. Two of the boxes are labeled "Correct Decision"; label the other two boxes Type I Error or Type II Error as appropriate. (8 points)

	H_0 True	H_0 False
Reject H_0	Type I error	Correct Decision
Fail to Reject H_0	Correct Decision	Type II Error

13. In a survey conducted by the American Animal Hospital Association, 37% of respondents stated that they talk to their pets on the answering machine or telephone. A vet found this hard to believe so he questioned 150 pet owners and discovered that 54 of them spoke to their pets on the answering machine or telephone. Does the vet have sufficient evidence to maintain his skepticism? (8 points)

1 PropZTest
$p_0 = .37$
$X = 54$
$n = 150$
Prop $\neq p_0$

$H_0: p = .37$
$H_a: p \neq .37$

$Z = -.2536$
$p = .7997$
fail to reject H_0

There is not sufficient evidence for the vet to maintain his skepticism

14. In 1990, the mean height of women 20 years of age or older was 63.7 inches based on data from the CDC, with a standard deviation of 3.4 inches. Suppose that a random sample of 45 women who are 20 years old or older today produced a mean of 63.9 inches. Is this strong evidence that women are taller now than 25 years ago? (8 points)

$H_0: \mu = 63.7$
$H_a: \mu > 63.7$
Ztest (Stats)
$\mu_0 = 63.7$
$\sigma = 3.4$ $\mu > \mu_0$
$\bar{x} = 63.9$
$n = 45$

$z = .3946$
$p = .346569 > .05$
fail to reject H_0
this is not sufficient evidence to think women are taller today

15. Below is a table of data for 16 patients before and after undertaking a particular diet plan. Conducted a paired t-test to determine if the diet had any effect on the dieters. (8 points)

	A	B
1	Before Diet	After Diet
2	213.4	200.1
3	225.0	216.4
4	217.0	195.6
5	183.7	175.0
6	197.2	201.3
7	223.6	214.8
8	224.2	215.7
9	215.2	200.7
10	202.4	211.7
11	217.7	216.1
12	221.0	208.5
13	219.9	188.4
14	205.4	206.4
15	195.1	180.9
16	218.0	184.1
17	207.6	202.3

enter data into L_1 & L_2
$L_1 - L_2 \rightarrow L_3$ = weight lost (neg is weight gain)

$H_0: d = 0$
$H_a: d > 0$
(since we want to lose weight on a diet)

TTest (Data)
$\mu_0 = 0$
List = L_3
Freq = 1
$\mu > \mu_0$

$t = 3.64$
$p = .0012 < .05$ reject H_0
yes, the diet does appear to be effective for losing weight

16. An experiment was conducted on patients with bipolar disorder. 55 patients received a new medication, while the control group of 60 patients received a placebo. Both patients were rated on the Young-Mania scale to measure their improvement. The experimental group had a mean improvement of 14.8 with a standard deviation of 12.5, while the control group had a mean improvement of 8.1 with a standard deviation of 12.7. Determine if the experimental group had a larger mean improvement than the control group with significance level $\alpha = 0.01$. (8 points)

2SampTTest (Stats)

$\bar{x}_1 = 14.8$
$S_{x_1} = 12.5$
$n_1 = 55$
$\bar{x}_2 = 8.1$
$S_{x_2} = 12.7$
$n_2 = 60$
$\mu_1 > \mu_2$
Pooled: No

$H_0: \mu_1 = \mu_2$
$H_a: \mu_1 > \mu_2$

$t = 2.849$
$p = .0026 < .01$
reject H_0
yes, there is sufficient evidence to think the drug is better than placebo

17. Use the data in the table below to conduct a two-sample proportion test to determine if there is sufficient evidence to think that rates of infection for those inoculated with the cholera vaccine were lower than the rate among those who were not inoculated. (8 points)

	infected	not infected	
inoculated	3	276	279
not inoculated	66	473	539
	69	749	818

Cholera Inoculation Study, 1894-96

Group1 → inoculated

Group2 → not inoculated

$H_0: p_1 = p_2$

$H_a: p_1 < p_2$

2PropZTest

$x_1 = 3$

$n_1 = 279$

$x_2 = 66$

$n_2 = 539$

$p_1 < p_2$

$z = -5.449$

$p = 2.53 \times 10^{-8} < .05$

reject H_0

yes, this is good evidence to think the vaccine is effective in reducing the number of cholera infections.