MAT 135, Discussion Questions 4.13

1. How would you describe the difference between a normal distribution and a Student t-distribution?

both are symmetric
T does not follow Empirical rule
bigger tails
depends on degrees of freedom (sample size)

2. What additional information is needed for a t-distribution?

degrees of freedom/sample size

3. What are some circumstances when we would use the t-distribution instead of the standard normal distribution?

Small sample sizes
when working w/ sample statistics and w/o known parameters

4. Interpret in words a 95% confidence interval for a mean of (50.1, 56.3) in a complete sentence.

we are 95% certain that the true mean of the population falls between 50.1 and 56.3.

5. Suppose that you have a mean distributed normally with a standard deviation of 14. How big a sample size is needed to estimate, with 95% confidence, the true value of the mean within one unit ($E = 1$).

$$n \geq \frac{z_{\alpha/2}^2 \sigma^2}{E^2} = \frac{1.96^2 * 14^2}{1} = 752.95$$

$\boxed{753}$ or more

6. The weights for a population of North American raccoons have a bell-shaped frequency curve with a mean of about 12 pounds and a standard deviation of about 2.5 pounds based on sample size of 68. Construct an 80% confidence interval and a 90% confidence interval. What do you notice about the two intervals?

Z int Stats	T-Int Stats	Z-Int Stats	T-Int Stats
$\sigma = 2.5$	$\bar{x} = 12$	$\sigma = 2.5$	$\bar{x} = 12$
$\bar{x} = 12$	$S_{\bar{x}} = 2.5$	$\bar{x} = 12$	$S_x = 2.5$
$n = 68$	$n = 68$	$n = 68$	$n = 68$
C-level: .8	Clevel: .8	C-level: .9	Clevel: .9
(11.611, 12.389)	(11.608, 12.392)	(11.501, 12.499)	(11.494, 12.506)

80% is narrower

7. In a simple random sample of 144 households in a county in Virginia, the average number of children in these households was 3.62 children. The standard deviation from this sample was 2.40 children. What is a 90% confidence interval for these results? What does it mean in the context of the problem?

Z-Int Stats	T-Int Stats
$\sigma = 2.4$	$\bar{x} = 3.62$
$\bar{x} = 3.62$	$S_x = 2.4$
$n = 144$	$n = 144$
C level: .9	C level: .9
(3.291, 3.949)	(3.2889, 3.9511)

We are 90% confident that the true mean # of children in this county in Virginia is between 3.3 and 3.9.

8. Suppose that a simple random sample of 100 men in Richmond were asked how much money they spent per visit at the barbershop. The responses resulted in a mean of \$21.43 and a standard deviation of \$7.84. Calculate a 95% confidence interval for these results.

Z-Int Stats	T-Int Stats
$\sigma = 7.84$	$\bar{x} = 21.43$
$\bar{x} = 21.43$	$S_x = 7.84$
$n = 100$	$n = 100$
C-level: .95	Clevel: .95
(19.893, 22.967)	(19.874, 22.986)

9. Redo the problem above but find a 99% confidence interval, and assume the data came from a sample size of 172 men. What do you notice about the two intervals? What can you conclude from this?

Z-Int Stats
$\sigma = 7.84$
$\bar{x} = 21.43$
$n = 100$
C-level: .99
(19.411, 23.449)

T-Int Stats
$\bar{x} = 21.43$
$S_x = 7.84$
$n = 100$
C-level: .99
(19.371, 23.489)

the interval is wider w/ more confidence

10. Which distribution did you choose for the example problems above? Why did you choose it? Redo the problem with the other distribution (use t if you used z before, or vice versa). What do you notice about the intervals?

See above for both

T-intervals are slightly wider

11. Calculate an 80% confidence interval for a sample with a mean of 54 and a standard deviation of 2.2. Assume the sample size is $n = 6$ and $n = 50$. Calculate the interval in each case with a z-score and a t-value. How do the results differ? What do you notice about the effect of the sample sizes and how it affects the results?

Z-Int Stats
$\sigma = 2.2$
$\bar{x} = 54$
$n = 6$
C-level: .8
(52.849, 55.151)
$n = 50$
(53.601, 54.399)

T-Int Stats
$\bar{x} = 54$
$S_x = 2.2$
$n = 6$
C-level: .8
(52.674, 55.326)
$n = 50$
(53.596, 54.404)

large sample sizes are more similar