

Instructions: Show all work. Some problems will instruct you to complete operations by hand, some can be done in the calculator. To show work on calculator problems, show the commands you used, and the resulting matrices. **Give exact answers** (yes, that means fractions, square roots and exponentials, and not decimals) unless specifically directed to give a decimal answer. This will require some operations to be done by hand even if not specifically directed to. Be sure to complete all parts of each question.

1. Find the determinant of the matrix $\begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 3 \\ 0 & 4 & -1 \end{bmatrix}$ by hand, first by the cofactor method, and then check it with the row-reducing method. (You must find the determinant by both methods to get full credit.)

Cofactor:

$$1 \begin{vmatrix} 2 & 3 \\ 4 & -1 \end{vmatrix} - 0 \begin{vmatrix} -1 & 3 \\ 0 & -1 \end{vmatrix} + 5 \begin{vmatrix} -1 & 2 \\ 0 & 4 \end{vmatrix} =$$

$$1(-2-12) + 0 + 5(-4-0) =$$

$$1(-14) + 5(-4) = -14 + (-20) = -34$$

row-reducing:

$$R_1 + R_2 \rightarrow R_2 \text{ (no change)}$$

$$\begin{vmatrix} 1 & 0 & 5 \\ 0 & 2 & 8 \\ 0 & 4 & -1 \end{vmatrix}$$

$$-2R_2 + R_3 \rightarrow R_3 \text{ (no change)}$$

$$\begin{vmatrix} 1 & 0 & 5 \\ 0 & 2 & 8 \\ 0 & 0 & -17 \end{vmatrix}$$

triangular, so product on the diagonal $(1)(2)(-17) = -34$

both methods agree

2. Use Cramer's Rule to find the solution of the system $\begin{cases} -5x_1 + 3x_2 = 9 \\ 3x_1 - x_2 = -5 \end{cases}$

$$A = \begin{bmatrix} -5 & 3 \\ 3 & -1 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ -5 \end{bmatrix}$$

$$\det A = 5 - 9 = -4$$

$$x_1 = \frac{\begin{vmatrix} 9 & 3 \\ -5 & -1 \end{vmatrix}}{-4} = \frac{-9 + 15}{-4} = \frac{6}{-4} = -\frac{3}{2}$$

$$x_2 = \frac{\begin{vmatrix} -5 & 9 \\ 3 & -5 \end{vmatrix}}{-4} = \frac{25 - 27}{-4} = \frac{-2}{-4} = \frac{1}{2}$$

$$\begin{bmatrix} -3/2 \\ 1/2 \end{bmatrix} = \vec{x}$$