Stat 2470, Quiz #4, Spring 2015 Name KEY

Instructions: Show all work. State any formulas used. If you use the calculator, you should say which function you used, and what you entered into it, as well as any output. I can only give partial correct for incorrect answers if I have something to grade.

1. For a card trick, a magician claims he can pull 3 aces out a deck in 4 cards or fewer. If he does not put the cards back, this situation can be modeled by a hypergeometric distribution. What is the probability he will succeed if he's really pulling the cards at random?

$N=52 \quad M=4 \quad n=4$

$$\frac{\binom{4}{3}\binom{48}{1}}{\binom{52}{4}} = 7.092\times10^{-4}$$

also + $\frac{\binom{4}{4}\binom{48}{0}}{\binom{52}{4}} = 3.697\times10^{-6}$ but adds negligibly to the result 7.129×10^{-4}

2. The number of customers arriving at an ice cream truck on a busy downtown street can be modeled with a Poisson distribution with a mean of 23 customers per hour. If the truck owner decides to take a 10-minute coffee break, what is the chance that he will have to turn away one or fewer customers?

$\mu = \frac{23}{6}$

$$p(x;\mu) = \frac{e^{-23/6}(23/6)^x}{x!} \quad x=0, x=1 : \frac{e^{-23/6}(23/6)^0}{0!} + \frac{e^{-23/6}(23/6)^1}{1!}$$

$= \text{poissoncdf}(23/6, 1) = .10458 \approx 10.5\%$

3. Find the value of k that will make $f(x) = \begin{cases} kx^{\frac{3}{2}}, 0 \le x \le 2 \\ 0, \quad otherwise \end{cases}$ a legitimate probability distribution.

$$\int_0^2 k x^{3/2}\,dx = k\left[\tfrac{2}{5}x^{5/2}\right]_0^2 = 1 \Rightarrow k\cdot\tfrac{2}{5}\left[2^{5/2}\right] = k\cdot\tfrac{2}{5}\sqrt{32}$$

$$k\cdot\tfrac{2}{5}\cdot 4\sqrt{2} = 1$$

$$\frac{8k\sqrt{2}}{5} = 1 \Rightarrow \boxed{k = \frac{5}{8\sqrt{2}}} \approx .44194$$