Stat 2470, Exam #1, Spring 2015 Name KEY

Instructions: Show all work. State any formulas used. If you use the calculator, you should say which function you used, and what you entered into it, as well as any output. I can only give partial correct for incorrect answers if I have something to grade.

1. Answer the following questions about the dataset shown here.

42	63	46	53	53	33	25	23	56	56
64	46	36	49	78	56	51	54	68	55
60	52	45	41	54	64	36	48	49	43

a. Create a stemplot for the data shown. Be sure that your plot has more than five stems. Describe the general shape of the graph, and include a key. (8 points)

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2 | 35
3 | 366
4 | 123566899
5 | 1233445666
6 | 03448
7 | 8
```

Key 4|5 = 45

roughly symmetric

b. Find the five-number summary. Determine if any of the datapoints in our set constitute outliers. Are they mild or extreme outliers? Use this information to sketch (to scale) a boxplot. (8 points)

both 23 and 78 are mild outliers

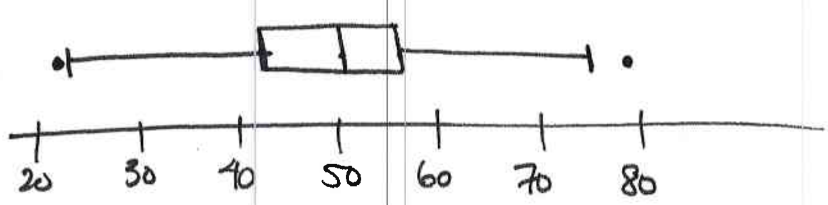
Min : 23

Q_1 : 43

Med : 51.5

Q_3 : 56

Max : 78



IQR = 13 1.5 * 13 = 19.5 43 − 19.5 = 23.5 56 + 19.5 = 75.5

c. What is the mean and the standard deviation of the data? How does the mean compare to the median? (5 points)

$\bar{X} = 49.967$

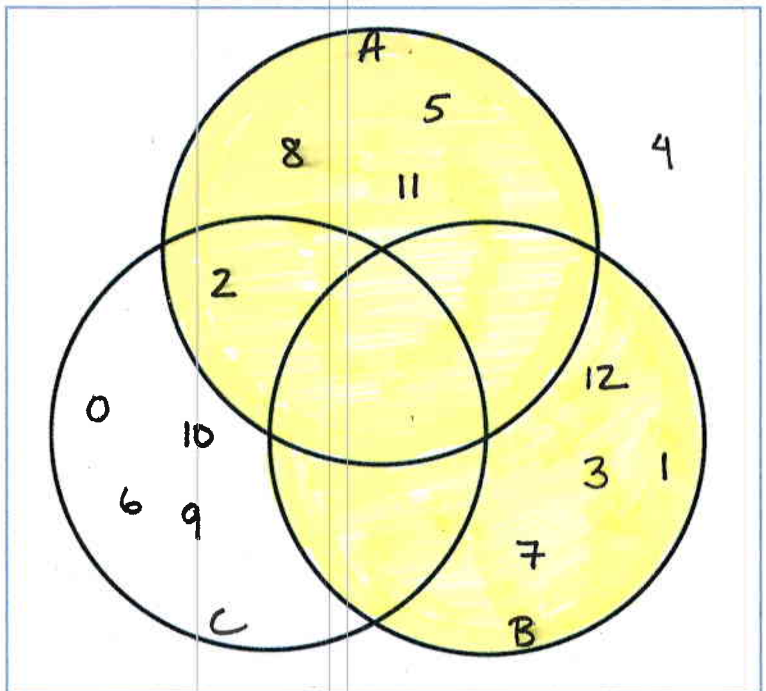
S = 12.104

mean is a bit lower than the median suggesting a possibly small left skew

2. Consider the universal set to be $\{0, 1, 2, \ldots, 10, 11, 12\}$. Let $A = \{2, 5, 8, 11\}$ and $B = \{1, 3, 7, 12\}$, and $C = \{0, 2, 6, 9, 10\}$. Draw a Venn diagram that illustrates each set. (5 points each)

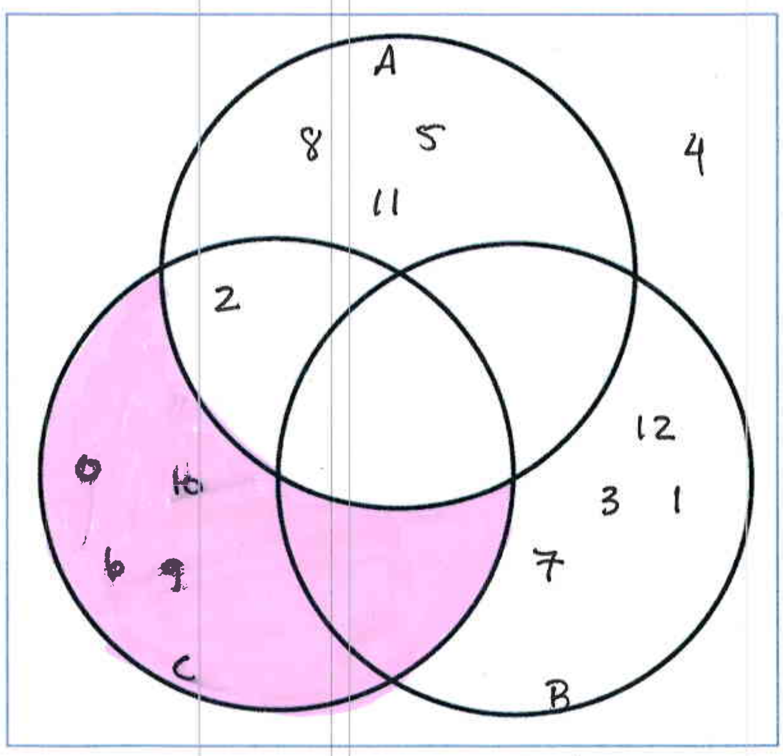
a. $A \cup B$

$\{1, 2, 3, 5, 7, 8, 11, 12\}$



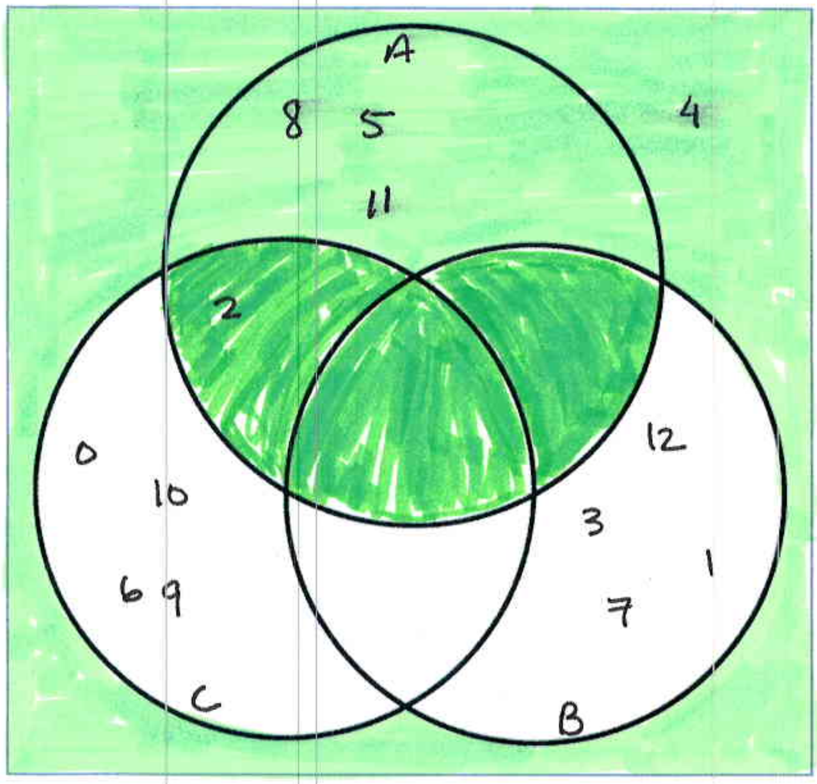
b. $A' \cap C$

$\{0, 6, 9, 10\}$



c. $(B' \cap C') \cup A$

$\{2, 4, 5, 8, 11\}$



3. Give the number of items in the indicated sample space. (4 points each)
   a. A bowl of marbles with 5 green marbles, 7 red ones, 11 white ones, 3 blue ones, and 9 yellow ones. We choose one marble.

      35

   b. A college establishes a committee composed of 6 members chosen from among the 300 faculty.

      $\binom{300}{6} = 9.628 \times 10^{11}$

   c. A raffle with first through fourth place prizes. If 200 tickets are sold, the ways that the prizes can be awarded.

      $200P4 = 1{,}552{,}438{,}800$

4. Suppose that a car can turn either left, right or go straight at a series of intersections. List the sample space of all possible outcomes for three turns in succession. (6 points)

LLL LLR LLS LRL LRS LRR LSL LSR LSS
RLL RLR RLS RRL RRS RRR RSL RSR RSS
SLL SLR SLS SRL SRS SRR SSL SSR SSS

5. Suppose that a certain factory divides its workers into three shifts: Day (45%), Swing (28%) and Night (27%). The manager did a survey of working conditions, and was told that 75% of the day shift liked working on the day shift and the rest wanted a transfer to another shift. Of the swing shift, 88% of the swing shift wanted to change shifts, and the rest liked it. Of the night shift, only 6% wanted to change shifts, and the rest wanted to stay. Use what you know about conditional probability to fill out the table below. (10 points)

	Day	Swing	Night	Total
Want to change shifts	11.25%	24.64%	1.62%	37.51%
Doesn't want to change shifts	33.75%	3.36%	25.38%	62.49%
Total	45%	28%	27%	100%

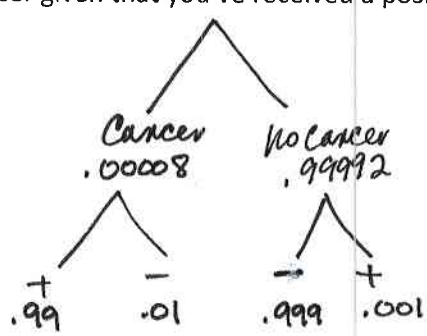
.75 * .45 = .3375

.88 * .28 = .2464

.06 * .27 = .0162

6. A certain type of cancer occurs in only 0.008% of the population. There is a blood test for this cancer which correctly detects the cancer 99% of the time, and correctly detects the absence of cancer 99.9% of the time. Use Bayes' Theorem and a tree diagram to find the probability of actually having cancer given that you've received a positive blood test. (7 points)

$\boxed{7.3\%}$



positive test:

$(.00008)(.99) + (.99992)(.001) =$

$.00107912$

$P(\text{cancer} \mid +) = \dfrac{(.00008)(.99)}{.00107912} = 7.3\%$

7. For each of the distributions given below, find the a) equation of the distribution, b) $P(X = 4)$, and c) $P(X \geq 2)$. (6 points each)

i. A binomial distribution with 25 trials, and $p = 0.6$

a) $b(x; 25, 0.6) = \binom{25}{x}(.6)^x(.4)^{25-x}$

b) $\binom{25}{4}(.6)^4(.4)^{21} = \text{binomialpdf}(25, .6, 4) = 7.21 \times 10^{-6}$

c) $1 - \text{binomialcdf}(25, .6, 1) = .999999957$

ii. A hypergeometric distribution with a population of $N = 500$, a sample size $n = 15$, and the number of success in the population is $M = 135$.

a) $\binom{135}{x}\binom{365}{15-x} / \binom{500}{15}$

b) $\binom{135}{4}\binom{365}{11} / \binom{500}{15} = .231$

c) $1 - \binom{135}{0}\binom{365}{15} / \binom{500}{15} - \binom{135}{1}\binom{365}{14} / \binom{500}{15} = 1 - .008 - .0474 = \boxed{.9442998}$

iii. A negative binomial distribution with $p = .75$, and requiring three successes.

a) $\binom{x+3-1}{3-1}(.75)^3(.25)^x = \binom{x+2}{2}(.75)^3(.25)^x$

b) $\binom{6}{2}(.75)^3(.25)^4 = .0247$

c) $1 - \binom{2}{2}(.75)^3(.25)^0 - \binom{3}{2}(.75)^3(.25)^1 = .2617$

iv. A Poisson distribution with $\mu = 5$.

a) $p(x; 5) = \dfrac{e^{-5}5^x}{x!}$

b) $p(4; 5) = \dfrac{e^{-5}5^4}{4!} = \text{poissonpdf}(5, 4) = .175467$

c) $1 - p(0; 5) - p(1; 5) = 1 - \text{poissoncdf}(5, 1) = .9595723$

8. Below is a discrete probability distribution. What is the expected value and the variance of the distribution. (7 points)

x	0	1	2	3	4	5
$p(x)$	0.1	0.2	0.25	0.2	0.15	0.1

$E(X) = 0(.1) + 1(.2) + 2(.25) + 3(.2) + 4(.15) + 5(.1) = \boxed{2.4}$

$E(X^2) = 0^2(.1) + 1^2(.2) + 2^2(.25) + 3^2(.2) + 4^2(.15) + 5^2(.1) = 7.9$

$V(X) = E(X^2) - [E(X)]^2 = 7.9 - 2.4^2 = \boxed{2.14}$

9. The number of car crashes at a particular intersection is 75 per year, and can be modeled as a Poisson process. What is the probability there will be more than 4 crashes at that same intersection in any given month? (6 points)

leave out 0,1,2,3 and 4

$\frac{75}{12} = 6.25$ Crashes per month

$$p(x;6.25) = \frac{e^{-6.25}(6.25)^x}{x!}$$

1 − poissoncdf(6.25, 4) = .747

10. Find the value of k that will make $f(x) = \begin{cases} k\sqrt[5]{x^2}, & 0 \le x \le 32 \\ 0, & otherwise \end{cases}$ a legitimate probability distribution. (7 points)

$$k\int_0^{32} x^{2/5}dx = k\frac{5}{7}x^{7/5}\Big|_0^{32} = k\cdot\frac{5}{7}\cdot 128 = 1 \qquad k = \frac{7}{640} \approx .0109375$$

$$f(x) = \frac{7}{640}x^{2/5} \qquad 0 \le x \le 32$$

11. Use the distribution in #10, and the value of k you found, to find the expected value of the distribution. (7 points)

$$\frac{7}{640}\int_0^{32} x\cdot x^{2/5}dx = \frac{7}{640}\int_0^{32} x^{7/5}dx = \frac{7}{640}\cdot\frac{5}{12}x^{12/5}\Big|_0^{32} =$$

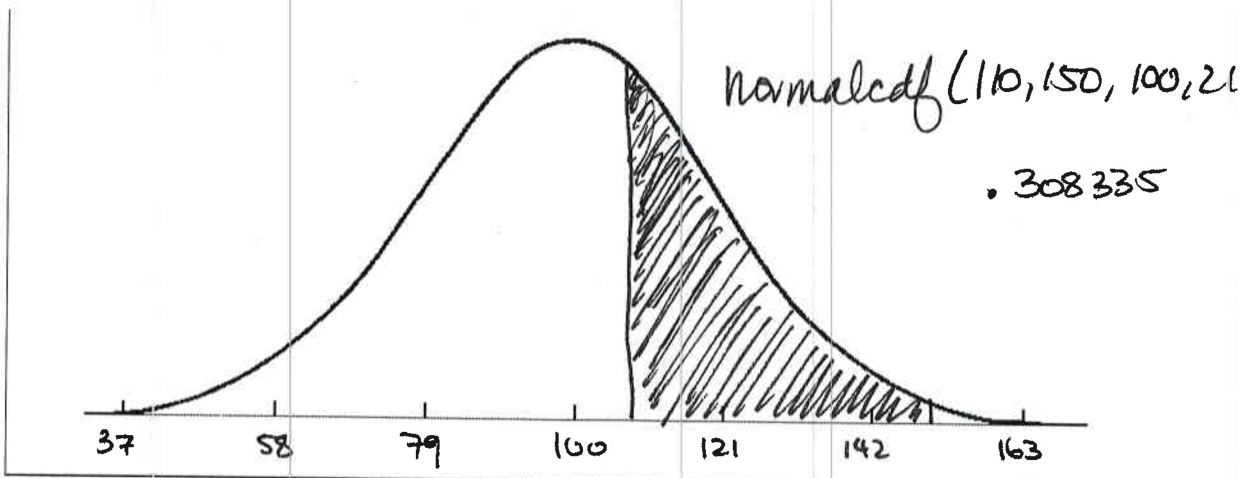
$$\frac{35}{7680}(4096) = \frac{56}{3} \approx 18.67$$

12. Use the probability distribution in #10, to find $P(0 \le X \le 4)$. (3 points)

$$\frac{7}{640}\int_0^{4} x^{2/5}dx \approx .0544$$

13. Find the value of x or z, or the indicated probability as needed and graph each situation on the normal curve shown. (4 points each)

a. $P(110 \leq X \leq 150), \mu = 100, \sigma = 21$



normalcdf(110, 150, 100, 21) =

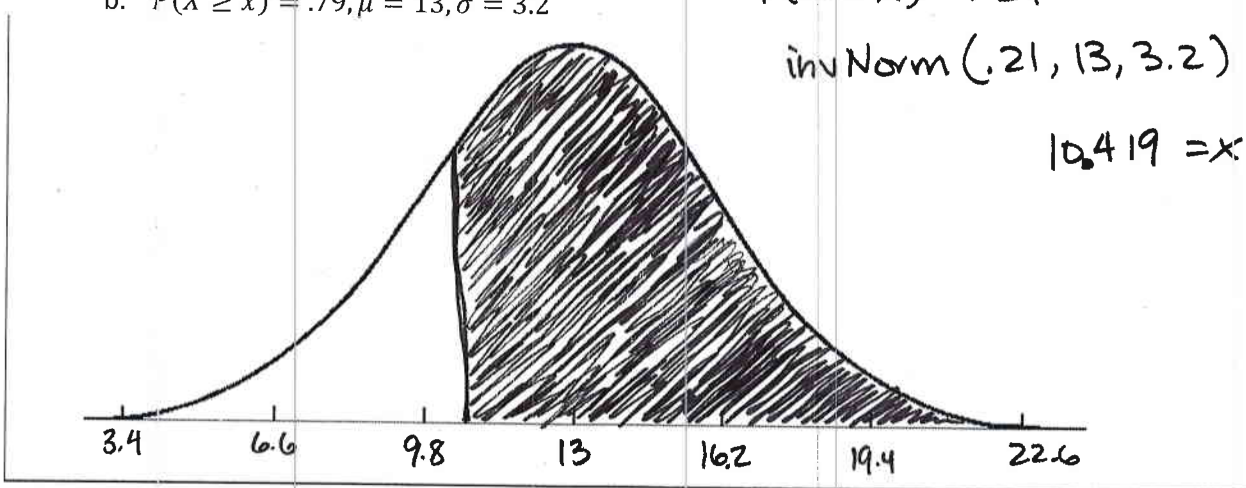
.308335

b. $P(X \geq x) = .79, \mu = 13, \sigma = 3.2$

$P(X \leq x) = .21$

invNorm(.21, 13, 3.2) =

10.419 = x



14. The gamma distribution is given by $f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}, x \geq 0 \\ 0, \quad \text{otherwise} \end{cases}$. Find the probabilities associated with the information given below. (5 points each)

a. $P(0 \leq X \leq 10), \alpha = 12, \beta = 3$

$\int_0^{10} \frac{1}{3^{12} \cdot 11!} x^{11} e^{-x/3} \, dx$

$= 1.87 \times 10^{-4}$

$\Gamma(12) = 11!$

$\Gamma(7) = 6!$

b. $P(X \geq 40), \alpha = 7, \beta = 11$

$\int_{40}^{\infty} \frac{1}{11^7 \cdot 6!} x^6 e^{-x/11} \, dx =$

$1 - \int_0^{40} \frac{1}{11^7 \cdot 6!} x^6 e^{-x/11} \, dx = 1 - .0763 = .923686$