MATH 1050 Course Review

***Directions**: Some of the questions on this review may require the use of the graphing calculator; others may require you to show all work. If an algebraic answer is required and work is not shown, you may not receive full credit on the final exam. On the final exam you must show work in the spaces provided and show graphs on the grids provided. Partial credit may be awarded on most problems. Reduce fractions to lowest terms. The final exam counts as 25% of your overall grade and contains 200 possible points. You will have 1 hour and 50 minutes to complete the final exam, but this review will most likely take you at least twice as long to complete.*

1. Given the following set of real numbers: $\{2,\ 5,\ 0,\ \pi,\ \frac{1}{3},\ 1.12,\ 1.\overline{3},\ 3.14,\ -13\}$

   List the numbers that are: a) rational b) natural c) integer

2. Given the following set of real numbers: $\left\{\frac{-2}{-3},\ \frac{5}{1.12},-2^2,\ \pi,\ \frac{15}{3},-7,\ 1.\overline{345},\ \frac{2\pi}{\pi},\ \frac{13}{-3}\right\}$

   List the numbers that are: a) positive b) negative

3. Plot the following real numbers on a number line: $\{3.14,\ 3.2,\ \frac{10}{3},\ \pi\}$

4. Which of the following expressions are equivalent to $-3^2+6(-1)^4$?

   a) $-3^2+(-6)^4$ b) -3^2+6-1^4 c) $-(3)^2+6(-1)^4$
   d) $(-3)^2+6(-1)^4$ e) -3^2-6 f) $-9+6$

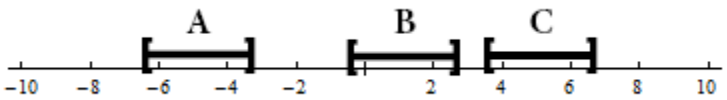
5. Which of the following expression are equivalent to $6-5(5-3)^2$?

   a) $6-(25-15)^2$ b) $6-5(5^2-3^2)$ c) $6+5(3-5)^2$
   d) $6-5(2)^2$ e) $6+5(-2)^2$ f) $6-5(4)$

6. Which of the following expressions are equivalent to $\frac{3}{7(5-4)}$?

   a) $\frac{3}{35-28}$ b) $\frac{3}{7}$ c) $\frac{3}{7}\cdot\frac{3}{(5-4)}$
   d) $\frac{3}{7\cdot5}-\frac{3}{7\cdot4}$ e) $\frac{1}{7}\cdot\frac{3}{(5-4)}$ f) $\frac{3}{7}\cdot\frac{1}{(5-4)}$

7. In which interval would your plot $\left(\frac{\sqrt{67{,}456}+123}{21-203}\right)^2$?



8. Evaluate algebraically. Verify the result using your calculator.

a) $\sqrt{\frac{34-5^2}{\sqrt{49}+7}}$ b) $6-5(5-3)^3$ c) $\frac{-2\,|7-16|\,\div 3+8}{\sqrt{36}+8\div 2^2}$

9. Which of the following expressions might you encounter when simplifying the expression $6+4[-2(5-3x)-(5x-3)]$ to $4x-22$?

a) $x=\frac{22}{4}$ b) $-22+4x$

c) $10[-2(5-3x)-(5x-3)]$ d) $6+4[-10+6x-5x+3]$

10. Which process shows a correct way to apply the distributive property to the expression $6\left[\frac{1}{2}(x+4)\right]$?

a) Process 1 b) Process 2 c) Neither are Correct d) Both are correct

<u>Process 1</u>:

$$6\left[\frac{1}{2}(x+4)\right]$$
$$=6\cdot\frac{1}{2}x+6\cdot 4$$
$$=3x+24$$

<u>Process 2</u>:

$$6\left[\frac{1}{2}(x+4)\right]$$
$$=6\left(\frac{x}{2}+2\right)$$
$$=3x+12$$

11. Evaluate $\frac{(4p+q)^2}{2p+q}$ for $p=5$ and $q=-4$. Express the answer as a fraction.

12. If $A=-2$, $B=\frac{1}{2}$, and $C=3$ then

a. determine the exact value of $\left(\frac{4-A+B(5-C)}{2C}\right)^2$

b. approximate the value of $\left(\frac{4-A+B(5-C)}{2C}\right)^2$ with a decimal approximation to 4 decimal places.

13. Is $g=0$ a solution to $3g-12=-3(4-g)$? Justify your answer.

14. Describe the solution set for each of the equations:

a) $-2(x+5)=5(1-x)+4(7-x)$ b) $\frac{2}{3}y-4=\frac{5}{2}y-\frac{3}{4}$

c) $\frac{m}{3}-2=\frac{m}{12}+\frac{m}{4}+5$ d) $4-8T+10=-2(4T-7)$

15. The formula $C = 4h + 9f + 4p$ describes the calorie count c for a serving of food in terms of h, the number of grams of carbohydrates, f, the number of grams of fat, and p, the number of grams of protein contained in the serving.

    a) Using this formula, create a formula describing the grams of fat per serving in terms of the calorie count, grams of carbohydrates, and grams of protein.
    b) If a serving of food has 346 calories, 46 grams of carbohydrates, and 18 grams of protein, determine the number of grams of fat in the serving

16. The surface area of a right circular cylinder, which has a top and a bottom, is given by the formula $S = 2\,\pi\,r^2 + 2\,\pi\,r\,h$ where r is the radius of cylinder and h is the height of the cylinder.

    a) What is the surface area of the cylinder if the radius is 2.7 meters and the height is 11.3 meters? Round your answer to the nearest hundredth.

    b) Using this formula, create a formula describing the height of the cylinder in terms of the surface area and the radius. (i.e. Solve the formula for h.)

    c) What is the height of the cylinder if the surface area is 890 ft^2 and the radius is 7 feet? Round your answer to the nearest hundredth.

17. Translate the following verbal descriptions into algebraic expressions:

    a) The perimeter of a rectangle whose width is half its length L.

    b) The sales tax on an item with a price of d dollars if the tax rate is 8%.

    c) The cost of y calculators if each calculator costs $89.95.

    d) Half the area of a rectangle whose length is half the width.

For problems 18 and 24, assign useful and descriptive variable names to represent each unknown, create a descriptive equation for each situation, use it to determine the unknown value(s), and state the solution(s) with correct units.

18. The width of a rectangle is 8 feet less than the length. If the perimeter of the rectangle is 64 feet, what are the dimensions of the rectangle?

19. A 45-foot rope is to be cut into three pieces. The second piece must be twice as long as the first piece and the third piece must be 9 feet longer than three times the length of the second piece. How long should each of the three pieces be?

20. Suppose Harry receives a 5% increase in his weekly salary. If his weekly salary after the increase is $475, what was his old salary before the increase?

21. A computer store just announced an 8% decrease in the price of their computers. If one particular computer model sells for $2075 after the decrease, find the original price of this computer. Round prices to the nearest cent

22. Find the measures of the angles of a triangle if the measure of one angle is twice the measure of a second angle and the measure of the third angle is 12° less than three times the measure of the second angle. Recall that the sum of the measures of the angles of a triangle is 180°.

23. Devon works for a software company and in 2006 earned a salary of $67,000. In 2008 Devon was laid-off, but found another job with a salary of $59,000. What percentage cut in salary did Devon experience?

24. Jason and Terelle are driving from Cleveland to Columbus. Jason leaves at 12:30pm travelling at 65mph and Terelle leaves at 1:00 travelling at 70mph. How much of a lead does Jason have when Terelle leaves Cleveland?

25. Given the sets of numbers below:

    a) { r | r is a real number between –3 and 7, including 7 }
    b) { x | x is a real number greater than or equal to 5 }

    Express both sets of numbers in each of the following forms:

    i) Set builder notation with inequality symbols
    ii) Graphed on a number line
    iii) Interval notation

26. Describe the solution set for the following linear inequalities using interval notation.

    a) $-12(x-5)-50 \geq 15$ b) $5+\frac{7w}{3} < 2-\frac{w}{2}$

27. Describe the solution set for the following linear inequalities using a graph (number line).

    a) $-12\left(\frac{y}{2}-5\right)-y \geq \frac{3-y}{2}$ b) $\frac{(7k-4)}{3} > 3-\frac{k-5}{2}$

28. What's wrong with this interval description? $(4,-\infty)$

29. What's wrong with this interval description? $[9,\infty]$

30. a) Is $(-5,-4)$ a solution to $2x-5y=10$? Justify your answer.

    b) List four other solutions to the equation $2x-5y=10$.

31. John has a job watering and tending gardens for people who are on vacation. He uses the formula $C = 6.25h$ to calculate the charge C for h hours spent watering and tending a garden. His price increases in 15-minute intervals with a 1-hour minimum.

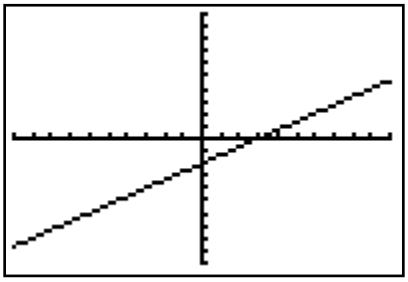
a) Complete the following table for the costs of jobs ranging from 1 to 3 hours.

Hours (h)	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
Charge (C)									

b) If a job that consists of tending a garden takes John 8 hours, how much does he charge?

32. Determine the intercepts of $y - 2x = 4$.

33. The intercepts of the line in the graph below are $(3, 0)$ and $(0, -2)$.



This graph represents the solutions to which of the following equations? Explain.

a) $y = \frac{2}{3}x - 2$

b) $y = 3x$

c) $y = 2x - 2$

d) $y = 2$

34. Draw a line that contains the point $(-4, 6)$ and has a slope of -8.

35. A line with no slope, or undefined slope, is different from a line with zero slope. Explain the difference.

36. Write the equation $\frac{3}{4}x + \frac{3}{5}y - 8 = 0$ in slope-intercept form. Then determine the slope and y-intercept.

37. The equation $S = 0.1p + 300$ models the weekly salary, S, of a salesperson who sells a total of p dollars of hardware each week. Both p and S are given in dollars.

a) Interpret the meaning of the ordered pair $(1000, 400)$.

b) If the salesperson sells a total of \$2350 of hardware in a week, what will their salary be?

c) A salesperson's salary was \$750 this week. How much hardware did they sell?

d) Graph the equation on a coordinate grid, using paper and pencil.

e) State a viewing window that would enable you to see a "complete" picture of this graph on your calculator.

38. Broyhill Furniture found that it takes 2 hours to manufacture each table for one of its special dining room sets. Each chair takes 3 hours to manufacture. A total of 1500 hours is available to produce tables and chairs of this style. The linear equation that models this situation is $2t + 3c = 1500$, where t represents the number of tables produced and $\boldsymbol{c}$ the number of chairs produced.

a) Complete the ordered pair solution $(0, \ )$ of this equation. Describe the manufacturing situation that this solution corresponds to.

b) Complete the ordered pair solution $(\ , 0)$ for this equation. Describe the manufacturing situation that this solution corresponds to.

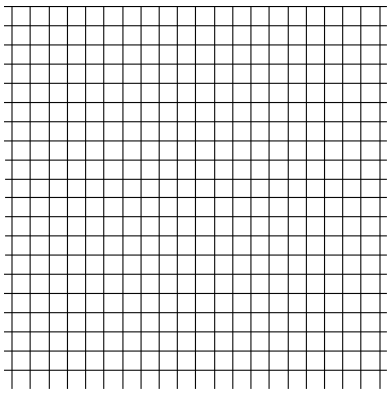
c) If 50 tables are produced, find the greatest number of chairs the company can make.

39. Which of the following equations are linear equations in the variables x and y?

a) $3x - 9y = 2$ b) $3x - 5^2y = 2$ c) $3x - 9y^2 = 2$
d) $3x - 9y - \sqrt{2} = 0$ e) $3 - 9y = \frac{1}{x}$ f) $Ax - B^2y = C$

40. Using a grid similar to the one provided below, draw a nice graph of the equations. "Nice" means the graph should show and label ALL important items and nothing more.

a) $\frac{-1}{4}x + 5y = 3$
b) $x = 7$
c) $y = 7$
d) $3a + 4b = 12$



41. A line passes through $(4, -8)$ and $(-4, -2)$. What is the slope of this line?

42. A line passes through $\left(4, \frac{-8}{3}\right)$ and $\left(\frac{-4}{5}, -2\right)$. What is the slope of this line?

43. If $(4,\ A)$ and $(5,\ B)$ are two points on the same line that has a positive slope then which is greater A or B?

44. If $(-4,\ A)$ and $(-5,\ B)$ are two points on the same line that has a positive slope then which is greater A or B?

45. Use slope to show that $(4, -8)$, $(1, -4)$, and $(-4, -2)$ cannot be on the same line.

46. Use slope to show that $(-4, -7)$, $\left(\frac{1}{2}, 2\right)$, and $(6, 13)$ are on the same line.

47. a) Write the equation of the horizontal line that passes through the point $(5, 3)$.

    b) Write the equation of the vertical line that passes through the point $(5, 3)$.

    c) Write the equation of a line that passes through the point $(5, 3)$ and has 0 slope.

    d) Write the equation of a line that passes through the point $(5, 3)$ and has no slope.

48. Based on the retirement plan available by his employer, Bill knows that if he retires after 20 years, his monthly retirement income will be \$3150. If he retires after 30 years, his monthly income increases to \$3600. Let x represent the number of years of service and y represent the monthly retirement income. Find the linear equation that relates the monthly retirement income to the number of years of service. Use the equation to predict the monthly income for 15 years of service.

49. Use the concept of slope to explain the difference between parallel and perpendicular lines.

50. $(2, 3)$ and $(5, -2)$ are on the same line. $(-1, -6)$ and (A, B) are on a different line of their own. These two lines are perpendicular. Determine possible values for A and B.

51. Approximate solutions to the system of equations $\begin{cases} 6x - y = -5 \\ 4x - 2y = 6 \end{cases}$ by the graphing method. Include a "nice" graph. A "nice" graph includes important points, useful points, axes with a scale, and omits extraneous information.

52. Use your calculator to approximate the solution to the systems of equations. Express your answer rounded to two decimal places.

a) $\begin{cases} y = -1.26x - 16.43 \\ y = 5.61x + 3.65 \end{cases}$ b. $\begin{cases} y = \frac{1}{-1.2}x - 34.7 \\ y = -\pi\, x + 3.65 \end{cases}$

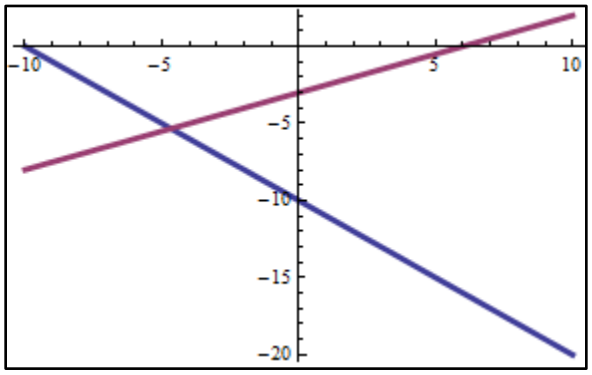
53. Algebraically solve the system of equations below using both the substitution and the elimination methods.

a) $\begin{cases} \frac{6}{5}x - y = -5 \\ 4x - \frac{2}{3}y = 3 \end{cases}$ b) $\begin{cases} 6x - \frac{2}{5}y = -5 \\ 4x - \frac{3}{4}y = \frac{3}{5} \end{cases}$ c) $\begin{cases} 6x - y = \frac{-5}{4} \\ 4x - 2^2y = 6 \end{cases}$ d) $\begin{cases} 6x - y = -5 \\ 4x - (-2)y = 6 \end{cases}$

54. Is the ordered pair $(-5, 6)$ a solution of the system: $\begin{cases} \frac{x}{15} + \frac{1}{3}y = 1 \\ 2x - \frac{y}{19} = 13 \end{cases}$? Explain

[Hint: You do not need to solve the system. Look at the signs in the second equation.]

55. Below is a graph of the lines determining this system of linear equations: $\begin{cases} Equation\ \#1 \\ Equation\ \#2 \end{cases}$



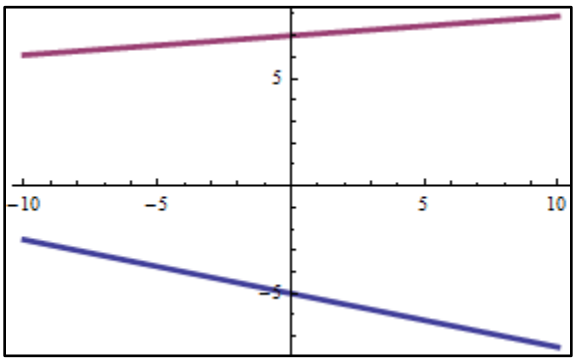
Could the ordered pair $(-15, 6)$ be a solution of the system? Explain your Thinking

56. Describe the three possible graphical representations of a linear system of two equations in two variables, and specify the number of solutions contained in each graphical representation.

57. How many solutions can a system of two linear equations have? Explain

58. Plot the point $(3, -4)$ on a coordinate plane. Draw two different lines that intersect at this point. Your picture represents a system of linear equations. Write the out the system in equation form.

59. Below is a graph of the lines determining this system of linear equations: $\begin{cases} Equation\ \#1 \\ Equation\ \#2 \end{cases}$



Which of the following are true?

a) $(-11, 1)$ seems like a reasonable solution to the system.
b) This system has no solutions.
c) The solution to this system would be represented by a point in the second quadrant.
d) It is impossible for this system to have an infinite number of solutions.
e) $(6, -5)$ is a good approximation to the solution to this system.

60. If $\begin{cases} Equation\ \#1 \\ Equation\ \#2 \end{cases}$ is a system of linear equations that has exactly one solution, then what can you say about the slopes of the two lines corresponding to these two equations?

61. If $\begin{cases} Equation\ \#1 \\ Equation\ \#2 \end{cases}$ is a system of linear equations that has no solutions, then what can you say about the slopes of the two lines corresponding to these two equations?

Solve problems 62-65 using a system of equations. Define descriptive and useful variables and write a system of equations that could be used to solve the problem, solve, and state the solutions precisely.

62. For this problem, define each variable used and write a system of equations that would describe the situation. **DO NOT SOLVE THE SYSTEM…JUST SET IT UP**

A chemist needs 210 milliliters of a 53% alcohol solution, but has only 41% and 72% alcohol solutions available. How many milliliters of each should be mixed to get the desired alcohol solution?

a) Let _____ represent _____
b) Let _____ represent _____
c) System of Equations _____
_____

63. James has available a 10% alcohol solution and a 60% alcohol solution. Find how many liters of each solution he should mix to make 50 liters of a 40% alcohol solution.

64. A hardware store stocks two types of hammers. The store has a total of 42 hammers, with sledge hammers selling at \$22.95 each and claw hammers selling at \$10.95 each. If the total value of the hammers is \$639.90, how many of each type are in stock?

65. Hertz car rental agency charges \$50 daily plus 10 cents per mile. Budget charges \$20 daily plus 25 cents per mile. Find the daily mileage for which the Budget charge for the day is the same as that of the Hertz charge for the day.

66. Graph the solution set for the system $\begin{cases} 6x - y \leq -5 \\ 4x - 2y > 6 \end{cases}$

67. Graph the solution set for the system $\begin{cases} 2A - B \leq -4 \\ 4A - 3B > 7 \end{cases}$

68. One vs. two variables
    a) If x is the only variable involved in the situation, then how would you graph $x \leq 5$?
    b) If x and y are two variables involved in the situation, then how would you graph $x \leq 5$?

69. Use exponent properties to represent the following expressions using only positive exponents:

a) $5w^0$ b) $(3xy^{-3})(-x^{-3}y^6)$ c) $\frac{9a^{-3}}{6} \cdot \frac{4}{a^2}$ d) $\frac{m}{s^2} \cdot s \cdot \frac{k}{s\,m}$ e) $\left(\frac{h^{-3}}{h^{-5}}\right)^{-2}$

70. What is the degree of this polynomial? $(-345 + y^2 - 52y)y^2 - (158y^3 + 45y - 871y^2 + 95)$

71. Write each polynomial as a sum of monomial terms (multiply out and collect like terms):

a) $(5x+3y)(x-2y)$ b) $(x-1)(x^2+x+1)$ c) $(5x-4y)^2$ d) $5x-6(4x+3)$ e) $(2m-3n)(2m+3n)$

72. Divide $15x^3 - 5x^2 + 10x$ by $5x^2$ using long division.

73. Reduce $\frac{15x^3-5x^2+10x}{5x^2}$, but leaving it in fraction form.

74. Divide $21x^2 + x - 10$ by $3x - 2$ using long division.

75. Reduce $\frac{21x^2+x-10}{3x-2}$, but leaving it in fraction form.

76. Divide $2x^3 + 3x - 4$ by $x + 2$ using long division.

77. Reduce $\frac{2x^3+3x-4}{x+2}$, but leaving it in fraction form.

78. When the calculator displays the result 3.234E–35, what number is being indicated?
    (i.e., how would you write the number on paper without listing all the zeros?)

79. Write 31,000,000,000 in scientific notation.

80. Write 3.1×10^{-4} in standard notation, without exponents.

81. Write the following expression in scientific notation. (Use a calculator to check the result.)

$$\frac{(2 \times 10^3)(3 \times 10^8)}{(8 \times 10^4)}$$

82. Factor completely:

a) $3y^2 - 8y - 3$ b) $2m^4 - 72m^2$ c) $k^3 - 8$ d) $x^2 - x - 42$

83. Write each polynomial as a product:

a) $3b + bx + 3y + xy$ b) $2A(A - B^2) - 5A(A - B^2)$ c) $9w^4 + 36w^2$ d) $49T^2 - 36$

84. Factor each polynomial:

a) $36G^2 - 60G + 25$ b) $27x^3 + 125y^3$

85. Factor out $h - 2$ from $h^3 + 4h^2 - 15h + 6$

86. Factor $3x^2y$ out from $3x^6y^5 + (-6)x^5y^4 + 15x^5y^3 - 18x^4y$

87. Factor $x^{\frac{1}{2}}$ from $x^3 + 4x^2 - 15x^{\frac{3}{2}} + 6x^{\frac{1}{2}}$

88. Show that -1 is not a solution of $3x^7 - x^5 + 2x^4 + 5x^2 = 13x - 2$

89. Determine a value for k, so that -1 is a solution of $3x^7 - x^5 + 2x^4 + 5x^2 = 13x + k$

90. Solve each equation algebraically:

a) $6y^2 - 15y = 0$ b) $(3A + 1)(2A - 5) = 39$ c) $6t^2 - 5t - 4 = 0$ d) $T^3 = 9T$

91. Determine all solutions of $3t^2 + (-5)t + 4 = 2t^2 - 4t + 10$

92. Describe the solution set of $\frac{t^2}{6} - \frac{t}{6} = 1$

93. An object is dropped off the top of a building that is 256 feet high. The distance, in feet, above the ground at s seconds is given by $H(s) = -16s^2 + 256$. Determine when the object hits the ground. Solve this problem both algebraically and using your calculator.

94. The shorter leg of a right triangle is 3 centimeters less than the other leg. Find the length of the two legs if the hypotenuse is 15 centimeters.

95. True or False
If you multiply two prime polynomials together, then the resulting polynomial is also prime. Explain.

96. True or False
If T and W are two polynomials, then the degree of their product $T \cdot W$ is the sum of their individual degrees

$$\text{degree of } T \cdot W = \text{degree of } T + \text{degree of } W$$