Stat 2470, Quiz #9, Spring 2014 Name KEY

Instructions: Show all work. You may use your calculator rather than compute formulas by hand, but if you do, 'show work' by saying which program you used to obtain the result and what information you entered. Round measures of center to one decimal place more than the data, and variance/standard deviation to two decimal places more than the original data. Round probabilities to three decimal places (or percent plus one decimal place).

1. Consider the joint probability distribution $p(x,y)$ shown here.

$p(x,y)$		y				
		0	1	2	3	
x	0	0.08	0.07	0.06	0.04	.25
	1	0.06	0.15	0.13	0.10	.44
	2	0.05	0.04	0.04	0.07	.2
	3	0.00	0.04	0.05	0.06	.15
		.19	.3	.28	.27	

Find the expected value of the distribution. $E(xy)$

$E(xy) = 0\cdot0\cdot\cancel{.08} + 0\cdot\cancel{1\cdot.07} + .06\cancel{\times0}\times2 + .04\cancel{\times0}\times3 + 0\times\cancel{1(.06)} + 1\times.15 + 2\times.13 + 3\times.10 + \cancel{0\times.05} + 2\times.04 + 4\times.04 + 6\times.07 + \cancel{0\times0} + 3\times.04 + 6\times.05 + 9\times.06 = 2.17$

$E(x) = \cancel{0\times.25} + 1\times.44 + 2\times.2 + 3\times.15 = 1.29$
$E(Y) = \cancel{0\times.19} + 1\times.3 + 2\times.28 + 3\times.27 = 1.67$

2. Consider the joint probability distribution $f(x,y) = 12x^3y^2, 0 \le x \le 1, 0 \le y \le 1$. Find the covariance of the distribution.

$E(xy) = \int_0^1\int_0^1 12x^4y^3\,dy\,dx = \int_0^1 3x^4\,dx = \frac{3}{5}$

$\int_0^1 12x^3y^2\,dy = f_x = 4x^3 \qquad \int_0^1 4x^4\,dx = \frac{4}{5} = E(x) = \mu_x$

$\int_0^1 12x^3y^2\,dx = f_y = 3y^2 \qquad \int_0^1 3y^3\,dy = \frac{3}{4} = E(y) = \mu_y$

$E(XY) - \mu_x\mu_y = \frac{3}{5} - \left(\frac{4}{5}\right)\left(\frac{3}{4}\right) = \frac{3}{5} - \frac{3}{5} = 0$

3. Explain the Central Limit Theorem in your own words.

means of samples from a distribution will cluster around the mean of the distribution w/ a normal distribution. for large enough samples, this will be true even if original distribution is not normal