

Instructions: Show all work. You may use your calculator rather than compute formulas by hand, but if you do, 'show work' by saying which program you used to obtain the result and what information you entered. Round measures of center to one decimal place more than the data, and variance/standard deviation to two decimal places more than the original data. Round probabilities to three decimal places (or percent plus one decimal place).

1. Consider the joint probability distribution  $p(x, y)$  shown here.

$p(x, y)$		$y$			
		0	1	2	3
$x$	0	0.08	0.07	0.06	0.04
	1	0.06	0.15	0.13	0.10
	2	0.05	0.04	0.04	0.07
	3	0.00	0.04	0.05	0.06

.25  
.44  
.20  
.15

a. What is  $P(X = 2)$ ?

.2

b. What is  $P(X = Y)$ ?

$.08 + .15 + .04 + .06 = .33$

c. Find  $p_X(x)$ , &  $p_Y(y)$ .

$$P_X(x) = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline & .25 & .44 & .20 & .15 \end{array}$$

$$P_Y(y) = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline & .19 & .3 & .28 & .27 \end{array}$$

d. What is  $p_{X|Y}$  for  $Y = 3$ ?

$$P_{X|Y=3} = \begin{array}{c|cccc} & 0 & 1 & 2 & 3 \\ \hline & .148 & .37 & .257 & .222 \\ \downarrow & & & & \\ & .04 & .10 & .07 & .06 \\ \hline & .27 & .27 & .27 & .27 \end{array}$$

2. Consider the joint probability distribution  $f(x, y) = 12x^3y^2, 0 \leq x \leq 1, 0 \leq y \leq 1$ .

a. Verify that this is a valid probability distribution.

$$\int_0^1 \int_0^1 12x^3y^2 dy dx = \int_0^1 4x^3 dx = 1 \quad \checkmark$$

b. What is  $P(Y \geq 1 - X)$ ?

$$\int_0^1 \int_{1-x}^1 12x^3y^2 dy dx = \int_0^1 4x^3 \Big|_{1-x}^1 dx =$$

$$\int_0^1 4x^3 - 4x^3(1-x)^3 dx = .97142857... = \frac{34}{35}$$

