Stat 2470, Exam #1, Spring 2014   Name KEY

Instructions: You may use your calculator for any functions the TI-84/83 model calculator is capable of using, such as probability distributions and obtaining graphs of any data. To show work on these problems, report the functions and their syntax as entered. Other things, such as integrating, must be done by hand unless specifically directed otherwise. Round means to one more place than the original data, variances and standard deviations to two more than the original data. Round probabilities to three significant figures or use exact values. In order to receive partial credit on any problem, you must show some work or I will have nothing to award partial credit on. Be sure to complete all the requested parts of each problem.

1. Consider the following observations on shear strength of a joint bonded in a particular manner:

30.0	4.4	33.1	66.7	81.5	22.2	40.4	16.4	73.7	36.6	109.9

a. Determine the value of the sample mean.

$\bar{x} = 46.81$

b. Determine the value of the sample median. Why is it so different from the mean?

$\tilde{x} = 36.6$ large values like 109.9

c. Calculate a trimmed mean by deleting the smallest and largest observations. What is the corresponding trimming percentage? How does the value of this $\bar{x}_{tr}$ compare to the mean and median?

$\bar{x}_{tr} = 44.51$ $\frac{1}{11} = 9.1\%$ trimmed

between mean $\bar{x}$ & median $\tilde{x}$

d. What is the variance of the sample?

$(32.01144966)^2 = 1024.73$

2. A sample of 20 glass bottles of a particular type was selected, and the internal pressure strength of each bottle was determined. Consider the following partial sample information:

Median = 202.2
lower fourth = 196.0 Q_1
Upper fourth = 216.8 Q_3

extreme outlier

Three smallest observations	125.8	188.1	193.7
Three largest observations	221.3	230.5	250.2 ← mild outlier

a. Are there any outliers in the sample?

$IQR = 20.8$

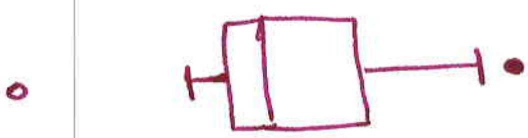
$1.5\ IQR = 31.2$

$3(IQR) = 62.4$

mild outliers $< 196 - 31.2 = 164.6$ or $> 216.8 + 31.2 = 248$

extreme outliers $< 196 - 62.4 = 133.6$ or $> 216.8 + 62.4 = 279.2$

b. Sketch a boxplot of the data.



3. Let A denote the event that the next item checked out at a college library is a math book, and let B be the event that the next item checked out is a history book. Suppose that P(A) = .40 and P(B) = .50. The probability that the book is a history of math book is $P(A \cap B) = 0.01$.

a. Why is it not the case that P(A) + P(B) = 1?

not all books are math or history

b. Calculate $P(A')$ $1 - .4 = .6$

c. Calculate $P(A \cup B)$.

$.4 + .5 - .01 = .89$

d. $1 - .89 = .11$

d. Calculate P($A' \cap B'$).

4. An Economics Department at a state university with five faculty members-Anderson, Box, Cox, Carter, and Davis-must select two of its members to serve on a program review committee. Because the work will be time-consuming, no one is anxious to serve, so it is decided that the representative will be selected by putting five slips of paper in a box, mixing them, and selecting two.

a. What is the probability that both Anderson and Box will be selected? (*Hint*: List the equally likely outcomes.)

$(\overset{3}{A}, \overset{6}{B})$ $(\overset{3}{A}, \overset{7}{C_1})$ $(\overset{3}{A}, \overset{10}{C_2})$ (A, D) (B, A) $(\overset{6}{B}, \overset{7}{C_1})$ (B, C_2)
(B, D) (C_1, A) (C_2, B), (C_1, C_2), (C_1, D) (C_2, A) (C_2, B)
(C_2, C_1) (C_2, D) (D, A) (D, B) (D, C_1) (D, C_2)

$P[(A, B) \text{ or } (B, A)] = 2/20 = .1$

b. What is the probability that at least one of the two members whose name begins with C is selected?

$P(\text{at least one } C) = \frac{14}{20} = .7$

c. If the five faculty members have taught for 3, 6, 7, 10, and 14 years, respectively, at the university, what is the probability that the two chosen representatives have at least 15 years' teaching experience at the university?

$P(\text{at least 15 years}) = 1 - P(\text{at most 14 years})$
$1 - P[(A, B), (B, A), (A, C_1), (A, C_2), (C_1, A), (C_2, A), (B, C), (C, B)]$
$= 1 - \frac{8}{20} = .6$

5. Suppose that an organization has 15 members, 7 of whom are male, and 8 of whom are female. Answer the following questions based on this information.

a. How many possible combinations of President, Vice President and Secretary are there?

$_{15}P_3 = 2730$

b. How many possible combinations of President, Vice President and Secretary are all women?

$_8P_3 = 336$

c. If the officers are chosen at random, what is the probability that the officers will all be women?

$$\frac{336}{2730} \approx .123 \text{ or } 12.3\%$$

d. If they wish to select a subcommittee to plan events for the coming year made up of four people, how many possible committees are there with exactly two men and two women?

$$_7C_2 \cdot {_8C_2}$$

$$\binom{7}{2}\binom{8}{2} = 588$$

6. Consider the following information: where A = {Visa Card}, B = {MasterCard}, P(A) = .5, P(B) = .4, and P(A∩ B) = .25. Calculate each of the following probabilities.

a. $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{.25}{.5} = .5$

b. $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.25}{.4} = .625$

c. Given that an individual is selected at random and that he or she has at least one card, what is the probability that he or she has a Visa card?

$$P(A \cup B) = .5 + .4 - .25 = .65$$

$$P(A | A \cup B) = \frac{P[A \cap (A \cup B)]}{P(A \cup B)} = \frac{P(A)}{P(A \cup B)} = \frac{.5}{.65} = .7692$$

7. Seventy percent of all vehicles examined at a certain emissions inspection station pass the inspection. Assuming that successive vehicles pass or fail independently of one another, calculate the following probabilities.

a. P(all of the next three vehicles inspected pass)

$$P(X=3) \quad (.7)^3 = .343$$

b. P(at least one of the next three inspected fail)

$1 - P(\text{all pass}) = 1 - .343 = .657$

8. What are the conditions for a Bernoulli random variable?

success or failure
fixed probability
independent trials

9. The pmf(pdf) for X = the number of major defects on a randomly selected gas stove of a certain type is

x	0	1	2	3	4
$P(x)$	.10	.15	.45	.25	.05

Compute the following:

a. $E(X)$

$\sum_{x=0}^{4} x\,p(x) = 0(.1) + (1)(.15) + 2(.45) + 3(.25) + 4(.05)$

$= 2.0$

b. $V(X)$ directly from the definition $V(X) = E[(X-\mu)^2] = \sum_{x=0}^{4} (x-2)^2 p(x)$

$= 2^2(.1) + 1^2(.15) + 0^2(.45) + 1^2(.25) + 2^2(.05) = 1.0$

c The standard deviation of X $\sqrt{V(X)} = 1.0$

c. $V(X)$ using the shortcut formula

$\sum_{x=0}^{\infty} x^2 p(x) - \mu^2 = 0^2(.1) + 1^2(.15) + 2^2(.45) + 3^2(.25) + 4^2(.05)$

$- 2^2 =$

$5.0 - 4.0 = 1.0$

10. Suppose that only 25% of all drivers come to a complete stop at an intersection having flashing red lights in all directions when no other cars are visible. What is the probability that, of 20 randomly chosen drivers coming to an intersection under these conditions,

   a. At most 6 will come to a complete stop? $p = .25 \quad n = 20$

   $P(X \le 6) = \text{binomialcdf}(20, .25, 6) = .78578...$

   b. Exactly 6 will come to a complete stop?

   $P(X = 6) = \text{binomialpdf}(20, .25, 6) = .168609...$

   c. At least 6 will come to a complete stop?

   $P(X \ge 6) = 1 - P(X \le 5) = 1 - \text{binomialcdf}(20, .25, 5)$

   $= .382827...$

   d. How many of the next 20 drivers do you expect to come to a complete stop?

   $E(X) = np = .25 * 20 = 5$

11. The number of tickets issued by a meter reader for parking-meter violations can be modeled by a Poisson process with a rate parameter of five per hour. $\mu = 5$

   a. What is the probability that exactly three tickets are given out during a particular hour?

   $P(X = 3) = \text{poissonpdf}(5, 3) = .14037...$

   b. What is the probability that at least three tickets are given out during a particular hour?

   $P(X \ge 3) = 1 - P(X \le 2) = 1 - \text{poissoncdf}(5, 2) = .8753479...$

c. How many tickets do you expect to be given during a 45-min period?

$5 \cdot \frac{3}{4} = 3.75$ tickets

12. “Time headway” in traffic flow is the elapsed time between the time that one car finishes passing a fixed point and the instant that the next car begins to pass that point. Let X = the time headway for two randomly chosen consecutive cars on a freeway during a period of heavy flow. The following pdf of X is

$$f(x) = \begin{cases} .15e^{-.15(x-.5)} & x \geq .5 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability that the time headway is

a. At most 6 seconds?

$$P(X \leq 6) = \int_{.5}^{6} .15e^{-.15(x-.5)}\,dx = -e^{-.15(x-.5)}\Big|_{.5}^{6}$$

$$= 1 - e^{.15(5.5)} = .562$$

b. At least 6 seconds?

$$P(X \geq 6) = 1 - P(X \leq 6) = 1 - .562 = .438$$

c. At most 5 seconds?

$$P(X \leq 5) = \int_{.5}^{5} .15e^{-.15(x-.5)}\,dx = -e^{-.15(x-.5)}\Big|_{.5}^{5}$$

$$= 1 - e^{-.15(5.0)} = .491$$

d. Between 5 and 6 seconds?

$$P(5 \leq X \leq 6) = .562 - .491 = .071$$

13. If X is a normal random variable with mean 85 and standard deviation 10, compute the following probabilities by standardizing.

$\frac{100-85}{10} = 1.5$

a. $P(X \leq 100)$

$P(Z \leq 1.5) = .9332$

normalcdf (-E99, 1.5)

$\frac{65-85}{10} = -2$

b. $P(65 \leq X \leq 100)$

normalcdf (-2, 1.5) =

$P(-2 \leq Z \leq 1.5) = .910442\ldots$

$\frac{70-85}{10} = -1.5$

c. $P(X \geq 70)$

$P(Z \geq -1.5)$

$= .9332$

14. Let X have a standard gamma distribution with $\alpha = 6$. Evaluate the following:

a. $P(X \leq 5)$ $\quad P(X \leq 5) = F(5;6) = .384$

b. $P(X > 8)$ $\quad 1 - P(X < 8) = 1 - F(8;6) = .191$

c. $P(3 \leq X \leq 8)$ $\quad F(8;6) - F(3,6) = .725$