Math 2153, Quiz #8, Spring 2014 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Find the gradient of the function $f(x,y) = xy(1-x^2-y^2)$. $= xy - x^3y - xy^3$

$\nabla f = \langle y - 3x^2y - y^3, x - x^3 - 3xy^2 \rangle$

2. Find the gradient of the function $f(x,y) = \sin(xy^2)$. Sketch key features of the gradient and the general direction of the gradient in each region. Use this information to sketch some level curves of the function.

$\nabla f = \langle \cos(xy^2)\cdot y^2, \cos(xy^2)\cdot 2xy \rangle$

$= 0$ $\quad \cos(xy^2)y^2 = 0 \Rightarrow \cos xy^2 = 0$ or $y^2 = 0$ $\quad y = 0$

$\cos(xy^2)(2xy) = 0 \Rightarrow \cos xy^2 = 0$ or $2xy = 0$ $\quad x=0, y=0$

$xy^2 = \pi/2, 3\pi/2 \Rightarrow y = \pm\sqrt{\pi/2x}, \pm\sqrt{3\pi/2x}$

$(-1,1) \to \langle \cos(-1), -2\cos(-1) \rangle \searrow$

$(-1,-1) \to \langle \overset{+}{\cos}(-1), 2\underset{-}{\cos}(-1) \rangle \nearrow$

$(1,1) \to \langle \overset{+}{\cos}(1), 2\overset{+}{\cos}(1) \rangle \nearrow$

$(1,-1) \to \langle \overset{+}{\cos}(1), -2\overset{+}{\cos}(1) \rangle \searrow$

graph on next page

$y = \pm\sqrt{\pi/2x}$ peak of wave

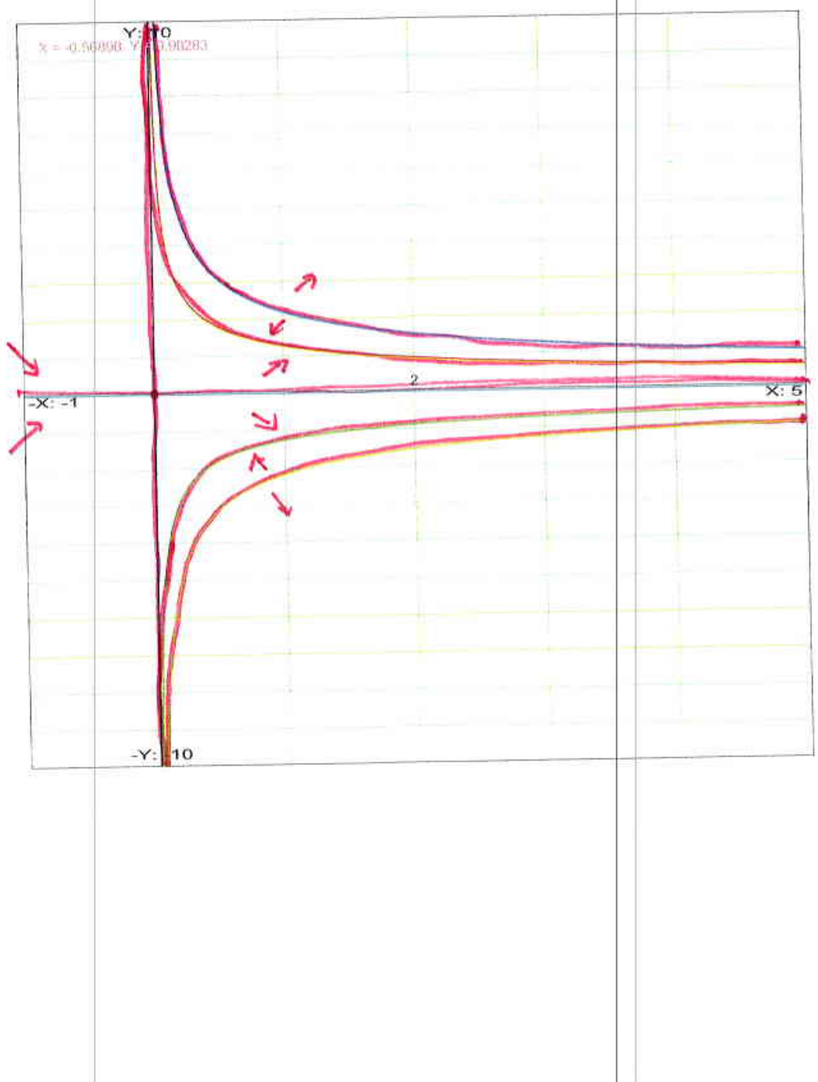
$y = \pm\sqrt{3\pi/2x}$ trough of wave

$(0,0)$ is a saddle point

3. Find $\nabla \times F$ for the vector field $F(x,y,z) = (3x^2y - z)\hat{\imath} + (yz + x^3)\hat{\jmath} + (\frac{1}{2}y^2 - x)\hat{k}$.

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2y - z & yz + x^3 & \frac{1}{2}y^2 - x \end{vmatrix} = \underbrace{(y-y)}_{0}\hat{\imath} - \underbrace{(-1-(-1))}_{0}\hat{\jmath} + \underbrace{(3x^2 - 3x^2)}_{0}\hat{k}$$

$\vec{\nabla} \times \vec{F} = \vec{0}$



these lines repeat all over the graph