

KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Find the total differential for  $z = ye^x$  at the point  $(0,1)$  and use this information to approximate the function at  $(0.1, 0.95)$ .

$$dz = ye^x dx + e^x dy$$

$$dz(0,1) = 1 dx + 1 dy$$

$$dz \approx 1(.1) + (1)(-.05)$$

$$= +.05$$

$$dx = .1$$

$$dy = -.05$$

$$z(0,1) + .05 = 1e^0 + .05 = 1.05$$

2. Integrate.  $\int_1^4 \int_1^{\sqrt{x}} 2ye^{-x} dy dx$

$$\int_1^4 y^2 e^{-x} \Big|_1^{\sqrt{x}} dx = \int_1^4 (x-1)e^{-x} dx$$

$$u = x-1 \quad dv = e^{-x} dx$$

$$du = dx \quad v = -e^{-x}$$

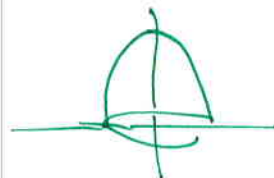
$$-(x-1)e^{-x} + \int e^{-x} dx = -(x-1)e^{-x} - e^{-x} \Big|_1^4$$

$$-3e^{-4} - e^{-4} - (0 - e^{-1})$$

$$-4e^{-4} + e^{-1} = \frac{1}{e} - \frac{4}{e^4}$$

3. Find the volume of the solid bounded by  $f(x,y) = -x^2 - y^2 + 9, z = 0$ . Set the integral up in rectangular and polar coordinates. Integrate the polar version.

$$\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (-x^2 - y^2 + 9) dy dx$$



$$dy dx = r dr d\theta$$

$$x^2 + y^2 = r^2 \Rightarrow$$

$$r = 3 \quad f(r, \theta) = -r^2 + 9$$

$$\int_0^{2\pi} \int_0^3 (-r^2 + 9) r dr d\theta = \int_0^{2\pi} \int_0^3 -r^3 + 9r dr d\theta =$$

$$\int_0^{2\pi} \left[ -\frac{r^4}{4} + \frac{9}{2}r^2 \Big|_0^3 \right] d\theta = \int_0^{2\pi} \left[ -\frac{81}{4} + \frac{81}{2} \right] d\theta = \left( \frac{81}{4} \right) \theta \Big|_0^{2\pi} =$$

$$\frac{81}{4} \cdot 2\pi = \frac{81\pi}{2}$$