Math 2153, Quiz #16, Spring 2014 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. An object is dropped into a gravity field with $\vec{a} = -5\hat{j}$ ft/sec^2. It has initial velocity $\vec{v}(0) = 2\hat{\imath} + \hat{j}$ and initial position $\vec{r}(0) = -\hat{\imath} + 300\hat{j}$. Find the position function for the particle at time t>0. When and where does the particle hit the ground?

$$\int -5\hat{j}\,dt = c_1\hat{i} + (5t + c_2)\hat{j} + c_3\hat{k} = 2\hat{i} + 1\hat{j} + 0\hat{k}$$

$c_1 = 2$ $\quad c_3 = 0$

$-5t + c_2 = 1$

$t = 0 \quad c_2 = 1$

$$\vec{v}(t) = 2\hat{i} + (-5t+1)\hat{j} + 0\hat{k}$$

$$\int 2\hat{i} + (-5t+1)\hat{j} + 0\hat{k}\,dt = (2t + c_1)\hat{i} + (-\tfrac{5}{2}t^2 + t + c_2)\hat{j} + c_3\hat{k}$$

$$= -\hat{i} + 300\hat{j} + 0\hat{k}$$

$t=0 \Rightarrow$ $c_1 = -1$ $\quad c_2 = 300$ $\quad c_3 = 0$

position $\boxed{\vec{r}(t) = (2t-1)\hat{i} + (\tfrac{5}{2}t^2 + t + 300)\hat{j}}$

2. Use Lagrange Multipliers to maximize the function $f(x, y) = x^2 + 2y^2 - x$ subject to the constraint $x^2 + y^2 = 1$.

$x^2 + y^2 - 1 = 0$

$f_x = \lambda g_x \rightarrow 2x - 1 = 2\lambda x \Rightarrow 2x - 2\lambda x = 1$

$2x(1-\lambda) = 1$

$1 - \lambda = \frac{1}{2x} \Rightarrow -\lambda = \frac{1}{2x} - 1$

$\lambda = 1 - \frac{1}{2x}$

$f_y = \lambda g_y \rightarrow 4y = 2\lambda y$

$\Rightarrow y = 0$

or $4 = 2\lambda$

$\Rightarrow \lambda = 2 \Longrightarrow 2 = 1 - \frac{1}{2x}$

$1 = -\frac{1}{2x}$

$2x = -1$

$x = -\frac{1}{2}$

$x^2 + y^2 - 1 = 0$

for $y = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

$(1,0)$ $(-1,0)$

for $x = -\frac{1}{2} \Rightarrow \frac{1}{4} + y^2 = 1 \quad y^2 = \frac{3}{4} \quad y = \pm\frac{\sqrt{3}}{2}$

$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$, $(-\frac{1}{2}, -\frac{\sqrt{3}}{2}) \Leftarrow$ both are maxima

$f(1,0) = 1 + 0 - 1 = 0$ $\quad f(-1,0) = 1 + 0 - (-1) = 2$ $\quad f(-\frac{1}{2}, \frac{\sqrt{3}}{2}) = \frac{1}{4} + 2(\frac{3}{4}) - (-\frac{1}{2}) = 9/4$

$f(-\frac{1}{2}, -\frac{\sqrt{3}}{2}) = \frac{1}{4} + 2(\frac{3}{4}) - (-\frac{1}{2}) = 9/4$