Math 2153, Quiz #14, Spring 2014 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Use Stokes' Theorem to find the value of the line integral $\int_C \vec{F}\cdot d\vec{r} = \int\int_S (\nabla\times\vec{F})\cdot\vec{N}dS$ for $\vec{F}(x,y,z) = z\hat{\imath} + y\hat{\jmath} + x\hat{k}$ around the curve of the intersection of $g(x,y,z) = 4 - x^2 - y^2$ and the plane z=1.

$1 = 4 - x^2 - y^2$

$x^2 + y^2 = 3$

$$\nabla\times F = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z & y & x \end{vmatrix} = 0\hat{\imath} - (1-1)\hat{\jmath} - 0\hat{k} = \vec{0}$$

$$\iint_R \vec{0}\cdot N dS = 0$$

$$\int_0^{2\pi}\int_0^{\sqrt{3}} \vec{0}\cdot\langle 2x, 2y, -1\rangle dA = 0$$

2. Find the implicit partial derivative $\frac{\partial z}{\partial x}$ for the implicit function $x\ln y - x^2 z + e^{yz} = 0$

$$-\frac{F_x}{F_z} = -\frac{\ln y - 2xz}{-x^2 + ye^{yz}} \quad \text{or}$$

$$\ln y - 2xz - x^2 z_x + e^{yz} y z_x = 0$$

$$\ln y - 2xz = (x^2 - e^{yz}y) z_x$$

$$z_x = \frac{\ln y - 2xz}{x^2 - ye^{yz}}$$

3. Use the chain rule to find the partial derivatives $\frac{\partial w}{\partial t}$ and $\frac{\partial w}{\partial s}$ for $w = xyz - y\sin x - \ln z$ where $x = t - s, y = t^2, z = se^t$.

$$\frac{\partial w}{\partial x} = yz - y\cos x = t^2 se^t - t^2\cos(t-s)$$

$$\frac{\partial w}{\partial y} = xz - \sin x = (t-s)se^t - \sin(t-s)$$

$$\frac{\partial w}{\partial z} = xy - \frac{1}{z} = (t-s)t^2 - \frac{1}{se^t}$$

↓

3 cont'd

$$\frac{\partial x}{\partial t} = 1 \qquad \frac{\partial x}{\partial s} = -1$$

$$\frac{\partial y}{\partial t} = 2t \qquad \frac{\partial y}{\partial s} = 0$$

$$\frac{\partial z}{\partial t} = se^t \qquad \frac{\partial z}{\partial s} = e^t$$

$$\frac{\partial w}{\partial t} = [t^2se^t - t^2\cos(t-s)](1) + [(t-s)se^t - \sin(t-s)]\,2t$$
$$+ \left[(t-s)t^2 - \frac{1}{se^t}\right]\cdot se^t$$

$$\frac{\partial w}{\partial s} = [t^2se^t - t^2\cos(t-s)](-1) + 0 + \left[(t-s)t^2 - \frac{1}{se^t}\right]e^t$$