Math 2153, Quiz #13, Spring 2014 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

$x^2 - xy \quad zy - \frac{1}{2}y^2 \quad zy - z^3$

1. Find the value of the work done in the vector field $\vec{F}(x,y,z) = (2x-y)\hat{\imath} + (z-y)\hat{\jmath} + (y-3z^2)\hat{k}$ on the path $\vec{r}(t) = t^2\hat{\imath} - t\hat{\jmath} + 3t\hat{k}$ on the interval $[0,1]$. If the field is conservative, use the fundamental theorem of line integrals.

$\int_C \vec{F}\cdot d\vec{r} =$ $\qquad r'(t) = 2t\hat{\imath} - \hat{\jmath} + 3\hat{k}$

$\int_0^1 \left[(2t^2+t)2t - (3t+t) + 3(-t - 27t^2)\right] dt =$

$\int_0^1 4t^3 + 2t^2 - 4t - 3t - 81t^2\, dt = \int_0^1 4t^3 - 79t^2 - 7t\, dt =$

$t^4 - \frac{79}{3}t^3 - \frac{7}{2}t^2\Big|_0^1 = 1 - \frac{79}{3} - \frac{7}{2} = \frac{-173}{6}$

field is not conservative

2. Use Green's Theorem to find the value of the line integral $\int_C (x-y)dx + (2x-3x)dy$ around the boundary of the rectangle with the vertices (0,0), (3,0), (3,4), (0,4), clockwise.

typo should be $(2x-3y)$

$\int_0^3\int_0^4 2-(-1)\, dy\, dx = 12(3) = 36$

as written though

$\int_0^3\int_0^4 -1-(-1)\, dy\, dx = 0$

3. Find the value of the surface integral $\int\int_S \vec{F}\cdot\vec{N}dS$ for the function $g(x,y,z) = x+y+z$ on the sphere $x^2+y^2+z^2 = 1$. (Use the back of the page.)

$F = \nabla g = \langle 1,1,1\rangle$

$\vec{N} = \langle 2x, 2y, 2z\rangle \qquad z = \sqrt{1-x^2-y^2}$

$\iint_S 2x + 2y + 2\sqrt{1-x^2-y^2}\, dA = \int_0^{2\pi}\int_0^1 2r^2\cos\theta + 2r^2\sin\theta + 2r\sqrt{1-r^2}\, dr\, d\theta$

$$\int_0^{2\pi}\int_0^1 2r^2\cos\theta + 2r^2\sin\theta + 2r\sqrt{1-r^2}\,dr\,d\theta$$

$u = 1-r^2$

$du = -2r\,dr$

$-\frac{1}{2}du = r\,dr$

$-\frac{1}{2}\int 2u^{1/2}\,du$

$$\int_0^{2\pi} \frac{2}{3}r^3\cos\theta + \frac{2}{3}r^3\sin\theta - \frac{2}{3}(1-r^2)^{3/2}\Big|_0^1 \,d\theta$$

$$\int_0^{2\pi} \frac{2}{3}\cos\theta + \frac{2}{3}\sin\theta - 0 + 0 + 0 + 2/3(1)^{3/2}\,d\theta$$

$$\int_0^{2\pi} \frac{2}{3}\cos\theta + \frac{2}{3}\sin\theta + \frac{2}{3}\,d\theta$$

$$\frac{2}{3}\sin\theta - \frac{2}{3}\cos\theta + \frac{2}{3}\theta\Big|_0^{2\pi}$$

$$0 - \cancel{\frac{2}{3}(1)} + \frac{2}{3}(2\pi) - 0 + \cancel{\frac{2}{3}(1)} + 0$$

$$\boxed{\frac{4\pi}{3}}$$