Math 2153, Quiz #11, Spring 2014 Name KEY

Instructions: Show all work. Use exact answers unless specifically asked to round. Be sure to complete all parts of each question.

1. Find the directional derivative of the function $f(x,y) = x\sin y - e^{xy}$ at the point $\left(1,\frac{\pi}{2}\right)$, in the direction of $\vec{u} = 5\hat{\imath} - 8\hat{\jmath}$.

$\|\vec{u}\| = \sqrt{25+64} = \sqrt{89}$

$\hat{u} = \frac{5}{\sqrt{89}}\hat{\imath} - \frac{8}{\sqrt{89}}\hat{\jmath}$

$\nabla f = \langle \sin y - ye^{xy}, x\cos y - xe^{xy} \rangle$

$\nabla f(1,\pi/2) = \langle 1 - \pi/2\, e^{\pi/2}, -e^{\pi/2} \rangle$

$\vec{\nabla} f \cdot \hat{u} = \langle 1 - \pi/2\, e^{\pi/2}, -e^{\pi/2} \rangle \cdot \left\langle \frac{5}{\sqrt{89}}, -\frac{8}{\sqrt{89}} \right\rangle =$

$\frac{5\left(\frac{2}{2}1 - \pi/2\, e^{\pi/2}\right)}{\sqrt{89}} + \frac{2 \cdot 8e^{\pi/2}}{2\cdot\sqrt{89}} = \frac{10 + (16 - 5\pi)e^{\pi/2}}{2\sqrt{89}} \approx .6044$

2. Find the equation of the tangent plane for $f(x,y) = x^2y - xy^3$, at the point (1,-2).

$F(x,y,z) = x^2y - xy^3 - z$

$\nabla F = \langle 2xy - y^3, x^2 - 3xy^2, -1 \rangle$

$\nabla F(1,-2) = \langle -4-(-8), 1-12, -1 \rangle = \langle 4, -11, -1 \rangle$

$z_0 = -2 - (-8) = 6$

$4(x-1) - 11(y+2) - (z-6) = 0$

3. Find the equation of the tangent plane for the parametric surface given by $\vec{r}(u,v) = u\cos v\,\hat{\imath} + u\sin v\,\hat{\jmath} + uv^3\hat{k}$ at the point $(-3, 0, -3\pi^3)$.

$\vec{r}_u = \cos v\,\hat{\imath} + \sin v\,\hat{\jmath} + v^3\hat{k}$

$\vec{r}_v = -u\sin v\,\hat{\imath} + u\cos v\,\hat{\jmath} + 3uv^2\hat{k}$

~~v = π, u = -3~~ ?

$v = -\pi$

$u = 3$

$uv^3 = -3\pi^3$

$u\sin v = 0$

$v = \pi, -\pi, 0,$ etc.

$u\cos v = -3$

$\cos(\pi) = -1$

$u = 3$

$\cos(-\pi) = -1$

$u = 3$

$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \cos v & \sin v & v^3 \\ -u\sin v & u\cos v & 3uv^2 \end{vmatrix} = (\cancel{3uv^2\sin v} - v^3u\cos v)\hat{\imath} - (3uv^2\cos v + \cancel{uv^3\sin v})\hat{\jmath} + \underbrace{(u\cos^2 v + u\sin^2 v)}_{= u}\hat{k}$

$= (-\pi)^3(3)(-1)\hat{\imath} - 3(3)(-\pi)^2(-1)\hat{\jmath} + 3\hat{k} = 3\pi^3\hat{\imath} + 9\pi^2\hat{\jmath} + 3\hat{k} \Rightarrow 3\pi^3(x+3) + 9\pi^2(y) + 3(z - 3\pi^3) = 0$