∇ Notation (Key)

1. $\nabla f = \langle y^2+2xz,\ 2xy+z^2,\ x^2+2yz \rangle$
2. $\nabla f = \langle y\cos z,\ x\cos z,\ -xy\sin z \rangle$
3. $\nabla f = \langle yze^{xy},\ xze^{xy},\ e^{xy} \rangle$
4. $\nabla f = \langle \sec^2(x+y),\ \sec^2(x+y) + z\sec^2(yz),\ y\sec^2(yz) \rangle$
5. $\nabla f = \langle \ln y,\ \frac{x}{y}+2yz,\ y^2+2z \rangle$
6. $\nabla f = \langle \frac{1}{2}(-10x)(25-5x^2-5y^2)^{-1/2},\ \frac{1}{2}(-10y)(25-5x^2-5y^2)^{-1/2},\ 0 \rangle$

$$= \left\langle \frac{-5x}{\sqrt{25-5x^2-5y^2}},\ \frac{-5y}{\sqrt{25-5x^2-5y^2}},\ 0 \right\rangle$$

7. $\nabla f = \langle -\frac{1}{2}(-2x)(1-x^2-y^2-z^2)^{-3/2},\ -\frac{1}{2}(-2y)(1-x^2-y^2-z^2)^{-3/2},\ -\frac{1}{2}(-2z)(1-x^2-y^2-z^2)^{-3/2} \rangle$

$$= \left\langle \frac{x}{(1-x^2-y^2-z^2)^{3/2}},\ \frac{y}{(1-x^2-y^2-z^2)^{3/2}},\ \frac{z}{(1-x^2-y^2-z^2)^{3/2}} \right\rangle$$

8. $\nabla f = \langle \csc\theta\cot\theta,\ -\csc^2\theta\cot\theta - \csc^3\theta,\ 0 \rangle$
9. $\nabla f = \langle 2r\cos 2\theta,\ -2r\sin 2\theta,\ 2z \rangle$
10. $\nabla f = \langle 2r\cos^2\theta,\ -2r\cos\theta\sin\theta,\ -1 \rangle$
11. $\nabla f = \langle 3r^2z + 6(1-r\cos\theta)^{-2}(-1)\cos\theta,\ +6(1-r\cos\theta)^{-2}(\sin\theta),\ r^3 \rangle$

$$= \left\langle 3r^2z - \frac{6\cos\theta}{(1-r\cos\theta)^2},\ \frac{6\sin\theta}{(1-r\cos\theta)^2},\ r^3 \right\rangle$$

12. $\nabla f = \langle e^{\theta},\ e^{\theta},\ 1 \rangle$
13. $\nabla f = \langle 4\cos\varphi,\ -4\sin\varphi,\ \frac{1}{\rho\sin\varphi}\cdot 0 \rangle = \langle 4\cos\varphi,\ -4\sin\varphi,\ 0 \rangle$
14. $\nabla f = \langle 3\csc\varphi\sec\theta,\ -3\csc\varphi\cot\varphi\sec\theta,\ \frac{1}{\rho\sin\varphi}3\rho\csc\varphi\sec\theta\tan\theta \rangle$

$$= \langle 3\csc\varphi\sec\theta,\ -3\csc\varphi\cot\varphi\sec\theta,\ 3\csc^2\varphi\sec\theta\tan\theta \rangle$$

15. $\nabla f = \langle 2\rho - 2\cos\varphi,\ 2\sin\varphi,\ 0\rangle$

16. $\nabla f = \langle 2\rho\sin^2\varphi + z\tan\theta,\ 2\rho\sin\varphi\cos\varphi,\ 2\csc\theta\sec^2\theta\rangle$

17. $\nabla\times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xyz & x^2y & yz^2 \end{vmatrix} = (z^2-0)\hat{i} - (0-xy)\hat{j} + (2xy - xz)\hat{k}$

$\langle z^2, xy, 2xy - xz\rangle$

18. $\nabla\times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \cos xy & \sin xz & \tan y \end{vmatrix} = (\sec^2 y - x\cos xz)\hat{i} - (0-0)\hat{j} + (z\cos xz + x\sin xy)\hat{k}$

$= \langle \sec^2 y - x\cos xz,\ 0,\ z\cos xz + x\sin xy\rangle$

19. $\nabla\times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{yz}{\sqrt{1-x^2y^2}} & \frac{xz}{\sqrt{1-x^2y^2}} & \arcsin xy \end{vmatrix} = \left(\frac{x}{\sqrt{1-x^2y^2}} - \frac{x}{\sqrt{1-x^2y^2}}\right)\hat{i} - \left(\frac{y}{\sqrt{1-x^2y^2}} - \frac{y}{\sqrt{1-x^2y^2}}\right)\hat{j}$

$+ \left(\frac{z\sqrt{1-x^2y^2} - xz\frac{1}{2}(1-x^2y^2)^{-1/2}\cdot 2xy^2}{1-x^2y^2} - \frac{z\sqrt{1-x^2y^2} - yz\frac{1}{2}(1-x^2y^2)^{-1/2} 2x^2y}{1-x^2y^2}\right)\hat{k}$

$= \vec{0}$

20. $\nabla\times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^{-1} & y^{-1} & 0 \end{vmatrix} = (0)\hat{i} - (0)\hat{j} + (0)\hat{k}$

21. $\nabla\times F = \left\langle \frac{1}{r}\cdot 0 - 0,\ 0-0,\ \frac{1}{r}(2r\sec\theta - r^2\cos\theta)\right\rangle$

$= \langle 0, 0, 2\sec\theta - r\cos\theta\rangle$

22. $\nabla\times F = \left\langle \frac{1}{r}\cdot 1 - 0,\ \sec^2 z - 0,\ \frac{1}{r}\left(\arctan r - \frac{r}{1+r^2} - 0\right)\right\rangle$

$= \left\langle \frac{1}{r},\ \sec^2 z,\ \frac{\arctan r}{r} - \frac{1}{1+r^2}\right\rangle$

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23. $\nabla\times F = \left\langle \frac{1}{r}\cdot z\sec^2\theta - -r\sin z,\ 0-0,\ \frac{1}{r}(2r\cos z - 0)\right\rangle$

$= \left\langle \frac{z}{r}\sec^2\theta + r\sin z,\ 0,\ 2\cos z \right\rangle$

24. $\nabla\times F = \left\langle \frac{1}{\rho\sin\varphi}(\rho\sin\varphi\sin\theta + \rho\varphi\cos\varphi\sin\theta - 0),\ \frac{1}{\rho}\left(\frac{1}{\sin\varphi}\rho\sin\varphi\sin\theta - 2\rho\sin\varphi\sin\theta\right),\ \frac{1}{\rho}(2\rho\cos\varphi - \rho\cos\varphi\cos\theta)\right\rangle$

$= \langle \sin\theta + \varphi\cot\varphi\sin\theta,\ -3\sin\theta,\ 2\cos\varphi - \cos\varphi\cos\theta \rangle$

25. $\nabla\times F = \left\langle \frac{1}{\rho\sin\varphi}(\theta\sin\varphi + \varphi\theta\cos\varphi - 2\varphi^2\cos\theta\sin\theta),\ \frac{1}{\rho}\left(\frac{1}{\sin\varphi}\cdot 0 - \theta\sin\varphi\right),\ \frac{1}{\rho}(\varphi^2\cos^2\theta - 0)\right\rangle =$

$\left\langle \frac{\theta}{\rho} + \frac{\varphi\theta}{\rho}\cot\varphi - \frac{2\varphi^2}{\rho}\csc\varphi\cos\theta\sin\theta,\ \frac{\theta}{\rho},\ \frac{\varphi^2}{\rho}\cos^2\theta \right\rangle$

26. $\nabla\times F = \left\langle \frac{1}{\rho\sin\varphi}(\rho^2\sin^3(\varphi\theta) + 3\varphi\rho^2\sin^2(\varphi\theta)\cos(\varphi\theta)\theta - 0),\ \frac{1}{\rho}\left(\frac{1}{\sin\varphi}\cdot 0 + 3\rho^2\sin^3(\varphi\theta)\right),\ \frac{1}{\rho}(\ln\rho + 1 - 0)\right\rangle$

$= \left\langle \rho\csc\varphi\sin^3(\varphi\theta) + 3\varphi\rho\csc\varphi\sin^2(\varphi\theta)\cos(\varphi\theta),\ 3\rho\sin^3(\varphi\theta),\ \frac{\ln\rho}{\rho} + \frac{1}{\rho}\right\rangle$

27. $\vec{\nabla}\cdot F = yz + x^2 + 2yz = 3yz + x^2$

28. $\vec{\nabla}\cdot F = -y\sin(xy) + 0 + 0 = -y\sin(xy)$

29. $\vec{\nabla}\cdot\vec{F} = yz\left(-\frac{1}{2}\right)(-2xy^2)(1-x^2y^2)^{-3/2} + xz\left(-\frac{1}{2}\right)(-2x^2y)(1-x^2y^2)^{-3/2} + 0$

$= \frac{xy^3z + x^3yz}{(1-x^2y^2)^{3/2}}$

30. $\vec{\nabla}\cdot\vec{F} = -\frac{1}{x^2} - \frac{1}{y^2}$

31. $\vec{\nabla}\cdot\vec{F} = \frac{1}{r}\left(3r^2\sin\theta\right) + \frac{1}{r}\left(r\sec\theta\tan\theta\right) + 1 =$

$3r\sin\theta + \sec\theta\tan\theta + 1$

32. $\vec{\nabla}\cdot\vec{F} = \frac{1}{r}(\tan z) + \frac{1}{r}\cdot 0 + 0 = \frac{\tan z}{r}$

33. $\vec{\nabla}\cdot\vec{F} = \frac{1}{r}(\ln r + 1) + \frac{1}{r}(0) + \tan\theta = \frac{\ln r}{r} + \frac{1}{r} + \tan\theta$

34. $\vec{\nabla}\cdot F = \frac{1}{\rho^2}\left(3\rho^2\sin\varphi\cos\theta\right) + \frac{1}{\rho\sin\varphi}\left(2\rho\sin\varphi\cos\varphi\sin\theta\right) + \frac{1}{\rho\sin\varphi}(0)$

$= 3\sin\varphi\cos\theta + 2\cos\varphi\sin\theta$

35. $\vec{\nabla}\cdot\vec{F} = \frac{1}{\rho^2}(5\rho^4) + \frac{1}{\rho\sin\varphi}\left(2\theta\sin\varphi\cos\varphi\right) + \frac{1}{\rho\sin\varphi}\left(2\varphi^2\cos\theta\sin\theta\right)$

$= 5\rho^2 + \frac{2\theta}{\rho}\cos\varphi - \frac{2\varphi^2}{\rho}\csc\varphi\cos\theta\sin\theta$

36. $\vec{\nabla}\cdot\vec{F} = \frac{1}{\rho^2}(1) + \frac{1}{\rho\sin\varphi}\left(\rho^2\cos\varphi\sin^3(\varphi\theta) + \rho^2\sin\varphi\cdot 3\sin^2(\varphi\theta)\cos(\varphi\theta)\cdot\theta\right)$

$+ \frac{1}{\rho\sin\varphi}(0) =$

$\frac{1}{\rho^2} + \rho\cot\varphi\sin^3(\varphi\theta) + 3\rho\theta\sin^2(\varphi\theta)\cos(\varphi\theta)$

37. $\nabla^2 f = 2 + 2 + 2 = 8$

38. $\nabla^2 f = 0 + 0 + \cancel{xy\cos z} = -xy\cos z$

39. $\nabla^2 f = zy^2e^{xy} + zx^2e^{xy} = e^{xy}(y^2z + x^2z)$

40. $\nabla^2 f = 2\sec^2(x+y)\tan(x+y) + 2\sec^2(x+y)\tan(x+y) + 2z^2\sec^2(yz)\tan(yz)$

$+ 2y^2\sec^2(yz)\tan(yz)$

$= 4\sec^2(x+y)\tan(x+y) + (2z^2 + 2y^2)\sec^2(yz)\tan(yz)$

41. $\nabla^2 f = -\frac{x}{y^2} + 2z + z$

42. $\nabla^2 f = \frac{-\sqrt{5}}{\sqrt{5-x^2-y^2}} - \frac{\sqrt{5}\,x^2}{(5-x^2-y^2)^{3/2}} + \frac{-\sqrt{5}}{\sqrt{5-x^2-y^2}} - \frac{\sqrt{5}\,y^2}{(\sqrt{5-x^2-y^2})^3}$

$= \frac{-2\sqrt{5}}{\sqrt{5-x^2-y^2}} - \frac{\sqrt{5}(x^2+y^2)}{(5-x^2-y^2)^{3/2}}$

43. $\nabla^2 f = \frac{1}{(1-x^2-y^2-z^2)^{3/2}} + \frac{3x^2}{(1-x^2-y^2-z^2)^{5/2}} + \frac{1}{(1-x^2-y^2-z^2)^{3/2}} + \frac{3y^2}{(1-x^2-y^2-z^2)^{5/2}}$

$+ \frac{1}{(1-x^2-y^2-z^2)^{3/2}} + \frac{3z^2}{(1-x^2-y^2-z^2)^{5/2}} =$

$\frac{3}{(1-x^2-y^2-z^2)^{3/2}} + \frac{3(x^2+y^2+z^2)}{(1-x^2-y^2-z^2)^{5/2}}$

44. $\nabla^2 f = \frac{1}{r}\csc\theta\cot\theta + \frac{\cos^3\theta + 5\cos\theta}{r\sin^4\theta} + 0 = \frac{1}{r}\csc\theta\cot\theta +$

$\frac{1}{r}\cot^3\theta\csc\theta + \frac{5}{r}\cot\theta\csc^3\theta$

45. $\nabla^2 f = 4\cos(2\theta) - 4\cos 2\theta + 2 = 2$

46. $\nabla^2 f = 4\cos^2\theta - 4\cos^2\theta + 4 = 4$

47. $\nabla^2 f = \frac{3(3r^5\cos^3(\theta)z - 9r^4 z\cos^2\theta + 9r^3\cos(\theta)(z) - 3r^2 z + 2r\cos^2\theta + 2\cos\theta)}{(1-r\cos(\theta))^3}$

$\frac{+6(\cos\theta - r - r\sin^2\theta)}{r(1-r\cos\theta)^3} =$

$\frac{-9r^4 z\cos^3\theta + 27r^3 z\cos^2\theta - 27r^2 z\cos\theta + 9rz - 12}{(1-r\cos\theta)^3}$

48. $\nabla^2 f = \dfrac{e^\theta}{r} + \dfrac{e^\theta}{r} + 1 = \dfrac{2e^\theta}{r} + 1$

49. $\nabla^2 f = \dfrac{8\cos\varphi}{\rho} + -\dfrac{8\cos\varphi}{\rho} + 0 = 0$

$\frac{3}{\rho}\csc^3\varphi\sec\theta$

50. $\nabla^2 f = \frac{6}{\rho}\csc\varphi\sec\theta - \frac{3}{\rho}\csc^3\varphi\sec\theta + \frac{3}{\rho}\csc^3\varphi\tan^2\theta\sec\theta +$

$= \frac{3}{\rho}\csc\varphi\sec\theta\left[2 - \csc^2\varphi + \csc^2\varphi\tan^2\theta + \csc^2\varphi\sec^2\theta\right]$

51. $\nabla^2 f = \dfrac{6r - 4\cos\varphi}{\rho} + \dfrac{4\cos\varphi}{\rho}$

52. $\nabla^2 f = \dfrac{6\rho\sin^2\varphi\cos\theta + 2\sin\theta}{\rho\cos\theta} + 6\cos^2\varphi - 2 + \dfrac{4\sin\theta}{\rho\sin^2\varphi\cos^3\theta}$

$= 6\sin^2\varphi + \frac{2}{\rho}\tan\theta + 6\cos^2\varphi - 2 + \frac{4}{\rho}\csc^2\varphi\tan\theta\sec^2\theta$

$= 6 - 2 + \frac{2}{\rho}\tan\theta + \frac{4}{\rho}\csc^2\varphi\tan\theta\sec^2\theta$

$= 4 + \frac{2}{\rho}\tan\theta\left[1 + 2\csc^2\varphi\sec^2\theta\right]$

53. i. $fg = x^2yz + xy^2z + xyz^2$

$\nabla(fg) = \langle 2xyz + y^2z + yz^2, x^2z + 2xyz + xz^2, x^2y + xy^2 + 2xyz\rangle$

$(x+y+z)\langle yz, xz, xy\rangle + xyz\langle 1,1,1\rangle =$

$\langle xyz + xyz + y^2z + yz^2, x^2z + xyz + xyz + xz^2, x^2y + xy^2 + xyz + xyz\rangle$

$= \langle 2xyz + y^2z + yz^2, x^2z + 2xyz + xz^2, x^2y + xy^2 + 2xyz\rangle$

verify each like this for 53, 54 & 55.