Math 1152, Quiz #7, Spring 2014 Name KEY

Instructions: Show all work. Use *exact* answers unless specifically asked to round. You may check your answers in the calculator, but you must show work to receive credit.

1. A tank initially contains 800L of pure water. A mixture containing a concentration of 24g/L of sugar at a rate of 3 L/sec, and the well-stirred mixture leaves the tank at the same rate. Find an expression for the amount of sugar in the tank at any time t. What amount of sugar would be needed in the tank to maintain the same level of sugar in the tank over time?

$\frac{dA}{dt} = \frac{24g}{L}\cdot\frac{3L}{sec} - \frac{A}{800L}\cdot\frac{3L}{sec} = 72 - \frac{3A}{800} = -\frac{3}{800}(-19200 + A)$ $A(0)=0$

$\int \frac{dA}{A-19200} = \int -\frac{3}{800}dt \Rightarrow \ln|A-19200| = -\frac{3}{800}t + C \Rightarrow A - 19200 = A_0 e^{-\frac{3}{800}t}$

$A(t) = 19200 + A_0 e^{-\frac{3}{800}t} \Rightarrow 0 = 19200 + A_0 e^{0} \Rightarrow A_0 = -19200 \Rightarrow A(t) = 19200 - 19200 e^{-\frac{3}{800}t}$

19200 grams is the equilibrium

2. Solve the differential equation $\frac{dy}{dx} = 2x\sqrt{1-y^2}$ by separation of variables.

$\int \frac{dy}{\sqrt{1-y^2}} = \int 2x\,dx$ $\arcsin y = x^2 + C$

$y = \sin(x^2 + C)$

3. Find the parametric equation of the line passing through the points $(-4,5)$ and $(3,-7)$.

$\Delta x = 3-(-4) = 7$

$\Delta y = -7-5 = -12$

$x = 7t - 4$

$y = -12t + 5$